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DEVELOPMENT OF EDUCATIONAL THEORY DERIVED FROM THREE
EDUCATIONAL THEORY MODELS. FINAL REPORT.
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THE CONCERN OF THIS STUDY WAS TO SET FORTH HYPOTHESES ABOUT HUMAN BEHAVIOR, HUMAN KNOWLEDGE, AND MATERIAL CHARACTERISTICS, AS FACTORS INVOLVED IN EDUCATION, AND THEN TO ESTABLISH WAYS FOR RELATING THE HYPOTHESES TO EMPIRICAL DATA. THE APPROACH TAKEN WAS TO INTERRELATE SET THEORY, INFORMATION THEORY, AND GRAPH THEORY WITH GENERAL SYSTEMS THEORY. USING THIS INTERRELATING PROCEDURE, THE INVESTIGATORS DEVELOPED A SYSTEM (THEORY MODEL) FOR RETRODUCING SCIENTIFIC EDUCATIONAL THEORY. (THEORY RETRODUCTION IS THE PROCESS OF ADDING CONTENT TO AN EXISTING THEORY MODEL TO FORM NEW THEORY.) THE NEWLY FORMED THEORY MODEL WAS THUS USED TO (1) DEFINE THE TERM, "SCHOOL," IN THE LANGUAGE OF A SYSTEM, (2) USE THE DEFINED SYSTEM TO GIVE MEANING TO THEORETICAL "SCHOOL PROPERTIES," AND (3) RELATE THESE "PROPERTIES" (FORM HYPOTHESES ABOUT "SCHOOL") TO CONSTITUTE EDUCATIONAL THEORY. PROCEDURES FOR RELATING THE THEORY TO DATA WERE OUTLINED, AND A PROJECTION WAS MADE ON HOW THE THEORY COULD BE EMPIRICALLY EVALUATED. (JH)

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DEVELOPMENT OF EDUCATIONAL THEORY DERIVED FROM THREE EDUCATIONAL THEORY MODELS

December 1966

U. S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

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DEVELOPMENT OF EDUCATIONAL THEORY DERIVED FROM THREE EDUCATIONAL THEORY MODELS.

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This research project report would not have been possible without the expertise of Grace Twine, Secretary of the Educational Theory Center, and the services of Verna Fullen and her staff in the Duplicating Center.

To Miss Henrietta, and Master Edwin Dodgson.

"Ch. Ch., Jan. 31st.

"My dear Henrietta,
"My dear Edwin,

"I am very much obliged by your nice little birthday gift--it was much better than a cane would have been--I have got it on my watch-chain, but the Dean has not yet remarked it.

"My one pupil has begun his work with me, and I will give you a description how the lecture is conducted. It is the most important point, you know, that the tutor should be dignified and at a distance from the pupil, and that the pupil should be as much as possible degraded.

"Otherwise, you know, they are not humble enough.

"So I sit at the further end of the room; outside the door (which is shut) sits the scout: outside the outer door (also shut) sits the sub-scout: half-way downstairs sits the sub-sub-scout; and down in the yard sits the pupil.

"The questions are shouted from one to the other, and the answers come back in the same way--it is rather confusing till you are well used to it. The lecture goes on something like this:--

"Tutor. What is twice three?
"Scout. What's a rice tree?
"Sub-Scout. When is ice free?
"Sub-sub-Scout. What's a nice fee?
"Pupil (timidly). Half a guinea!
"Sub-sub-Scout. Can't forge any!
"Sub-Scout. Ho for Jinny!
"Scout. Don't be a ninny!
"Tutor (Looks offended, but tries another question). Divide a hundred by twelve!
"Scout. Provide wonderful bells!
"Sub-Scout. Go ride under it yourself!
"Sub-sub-Scout. Deride the dunder-headed elf!
"Pupil (surprised). Who do you mean?
"Sub-sub-Scout. Doings between!
"Sub-Scout. Blue is the screen!
"Scout. Soup-tureen!

"And so the lecture proceeds.

"Such is Life.
"from
"Your most affect. brother,
"Charles L. Dodgson.

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INTRODUCTION

In Cooperative Research Project 1632[1], the retroductive method for constructing empirical educational theory from theory models was developed and tried out. Seven educational theory models were constructed and found to have heuristic utility in constructing educational theory. The logical next step was to concentrate on the construction of educational theory from one or more of these theory models. Among the seven educational theory models, it was noted that the one constructed from general systems theory provided a basic framework into which two of the others--one from information theory and one from graph theory--could be incorporated. The objective of this research, therefore, was the development of educational theory derived from these three educational theory models. In the course of the research, however, it was found that concepts from set theory were required. An extension of the project was granted, and the objective became the development of educational theory from an educational theory model constructed from set theory, information theory, graph theory, and general systems theory.

A resume of the report to follow indicates the procedure utilized in achieving the objective. First, the concepts of set theory, information theory, and graph theory had to be delineated and ordered. With respect to information and graph theory, this was an extension of educational theory models taken from the earlier project. In Chapters I through III, these results are presented. Next, the general systems educational theory model, also from the earlier project, was integrated with set theory, information theory, and graph theory and was extended. The SIGGS Theory Model resulted and is set forth in Chapter IV. To indicate the nature and uniqueness of the integration, in Chapter V there is an analysis of the literature and the SIGGS Theory Model with respect to the relation of set theory, information theory, and graph theory with general systems theory. In Chapter VI, to indicate the nature and uniqueness of the method of developing educational theory from the SIGGS Theory Model, the use in the literature of concepts incorporated in the SIGGS Theory Model is contrasted with the use in this project. The essence of the research, the educational theory, is the content of Chapter VII. Chapter VIII contains ways of relating the theory to data, so that the educational theory is seen to be more than sheer and idle speculation. Finally, in the Conclusion a projection to evaluate the educational theory is presented in order that a conclusion as to the adequacy of that theory may be forthcoming.

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[1] E. S. Maccia, G. S. Maccia, and R. E. Jewett, Construction of Educational Theory Models, Washington, D. C.: Office of Education, Department of Health, Education, and Welfare, 1963.

CHAPTER I

SET THEORY

Intuitive Explication of Set

Set theory is mathematical theory which characterizes sets. 'Set' is taken to be a primitive term. Set can be explicated intuitively by considering alternative referents. A set can be thought of as a collection, a class, an aggregate, a group, etc. As can be seen from these alternative referents, a set usually, although not always, has something within it which could be considered as belonging to the set: the objects of the collection, the members of the class, the points of the aggregate, the components of the group, etc. That which belongs to the set is called 'an element'. Moreover, the objects, members, points, components, etc. can themselves be taken as sets of elements; and if they are so taken, then the collection, the class, the aggregate, the group, etc. can be thought of as families of sets.

Notations

1. Lower case letters will be used as elements which are not considered themselves as having elements.[1]
2. Non-script upper case letters will be used as sets whose elements will be considered.[2]
3. Script upper case letters will be used as families of sets of the kind in 2.
4. $\in$ will be used for the elementhood relation.[3]

---

[1] Lower case letters are used also for functions.

[2] Underlined upper case letters are used for predicates.

[3] $\notin$ will be used to negate the elementhood relation.

4.1. $x \in X$, therefore, is read "an element, x, belongs to a set, X," or "x is an element of X".

5. $\{\ldots\}$ will be used for a set whose elements can be listed, where '...' refers to all the elements of the set.

6. $\{x \mid \underline{P}(x)\}$ will be used for a set whose elements, x, can be chosen and x makes the predicate, $\underline{P}$, true upon substituting x in $\underline{P}$.

Characterizations

1. Subset

1.1. $X \subset Y =_{Df} \forall x(x \in X \Rightarrow x \in Y)$

1.2. 'X is contained in Y' equals by definition 'for all x, x is an element of X only if x is an element of Y '.

2. Equals

2.1. $X = Y =_{Df} X \subset Y \land Y \subset X$

2.2. 'X is equal to Y' equals by definition 'X is contained in Y and Y is contained in X '.

2.3. $X \neq Y$ will be the negation of $X = Y$.

3. Union[4]

3.1. $\bigcup_{i \in I} X_i =_{Df} \{x \mid \exists i(i \in I \land x \in X_i)\}$

3.2. 'The union of X as i varies over I' equals by definition 'the set of x such that there is an i such that i is an element of I and x is an element of X_i '.

3.1a. $X_1 \cup X_2 =_{Df} \{x \mid x \in X_1 \lor x \in X_2\}$

3.2a. 'The union of X_1 and X_2' equals by definition 'the set of x such that x is an element of X_1 or x is an element of X_2 '.

---

[4] In definitions following this format: ___.1 is a generalized definition which is possible if one accepts the axiom of choice, while ___.1a and ___.1b constitute the inductive definition. The generalized definition permits union over a non-denumerable set of sets.

3.1b. $\bigcup_{i=1}^{n} X_i =_{Df} (\bigcup_{i=1}^{n-1} X_i) \cup X_n$

3.2b. 'The union of X where X is indexed from 1 to n' equals by definition 'the union of the union of X, where X is indexed from 1 to n minus 1, and X_n '.

4. Intersection

4.1. $\bigcap_{i \in I} X_i =_{Df} \{x \mid \forall i(i \in I \Rightarrow x \in X_i)\}$

4.2. 'The intersection of X as i varies over I' equals by definition 'the set of x such that for all i, i is an element of I only if x is an element of X_i '.

4.1a. $X_1 \cap X_2 =_{Df} \{x \mid x \in X_1 \wedge x \in X_2\}$

4.2a. 'The intersection of X_1 and X_2' equals by definition 'the set of x such that x is an element of X_1 and x is an element of X_2 '.

4.1b. $\bigcap_{i=1}^{n} X_i =_{Df} (\bigcap_{i=1}^{n-1} X_i) \cap X_n$

4.2b. 'The intersection of X where X is indexed from 1 to n' equals by definition 'the intersection of the intersection of X, where X is indexed from 1 to n minus 1, and X_n '.

5. n-tuple

5.1a. $(x_1,x_2) =_{Df} \{x_1,\{x_1,x_2\}\}$

5.2a. 'The ordered pair of x_1 and x_2' equals by definition 'the set of x_1 and the set of x_1 and x_2 '.

5.1b. $(x_1,x_2,\ldots,x_n) =_{Df} (x_1,x_2,\ldots x_{n-1}) \cup \{\{x_1,x_2,\ldots,x_n\}\}$

5.2b. 'The n-tuple of x_1, x_2, etc., and x_n' equals by definition 'the union of the n minus 1-tuple of x_1, x_2, etc., and x_{n-1} and the set of the set of x_1, x_2, etc., and x_n '.

6. Cartesian Product

6.1a. $X_1 \times X_2 =_{Df} \{(x_1,x_2) \mid x_1 \in X_1 \wedge x_2 \in X_2\}$

6.2a. 'The Cartesian product of X_1 and X_2' equals by definition 'the set of ordered pairs of x_1 and x_2 such that x_1 is an element of X_1 and x_2 is an element of X_2 '.

6.1b. $\prod_{i=1}^{n} X_i =_{Df} (\prod_{i=1}^{n-1} X_i) \times X_n$

6.2b. 'The Cartesian product of X where X is indexed from 1 to n' equals by definition 'the Cartesian product of the Cartesian product of X, where X is indexed from 1 to n minus 1, and X_n '.

7. Complement

7.1. $C_Y X =_{Df} \{x \mid x \in Y \wedge x \notin X\}$

7.2. 'The complement of X with respect to Y' equals by definition 'the set of x such that x is an element of Y and x is not an element of X '.

8. Subtraction

8.1. $Y - X =_{Df} C_Y X$

8.2. 'Y minus X' equals by definition 'the complement of X with respect to Y '.

9. Relation

9.1. $R =_{Df} \{(x_1,x_2,\ldots,x_n) \mid (x_1,x_2,\ldots,x_n) \in X \wedge X \subset \prod_{i=1}^{n} Y_i\}$

9.2. 'Relation' equals by definition 'the set of n-tuples of x_1, x_2, etc., and x_n, $(x_1,x_2,\ldots,x_n)$, such that $(x_1,x_2,\ldots,x_n)$ is an element of X, and X is contained in the Cartesian product of Y where Y is indexed from 1 to n '.

10. Equivalence

10.1. $X \sim Y =_{Df} \exists K(K \subset X \times Y \land \forall x(x \in X \Rightarrow \exists! y((x,y) \in K)) \land$

$\forall y(y \in Y \Rightarrow \exists! x((x,y) \in K)))$

10.2. 'X is equivalent to Y' equals by definition 'there is a K such that K is contained in the Cartesian product of X and Y, and for all x, x is an element of X only if there is a unique y such that the ordered pair of x and y, (x,y), is an element of K, and for all y, y is an element of Y only if there is a unique x such that (x,y) is an element of K '.

11. Cardinality

11.1. $n(X) =_{Df} \{x \mid x \in J_n \land J_n = \{y \mid y = 1, 2, \ldots, n \land n < \infty\} \land$

$J_n \sim X\}$

11.2. 'The cardinality of X' equals by definition 'the set of x such that x is an element of J_n, and J_n is equal to the set of y such that y is equal to 1, 2, etc., and n and n is less than infinity, and J_n is equivalent to X '.

12. Function

12.1. $\varphi \mid X \to Y =_{Df} \{(x,y) \mid \exists K((x,y) \in K \land K \subset X \times Y \land \forall v(v \in X \Rightarrow$

$\exists! u((v,u) \in K))\}$

12.2. 'The function, φ, defined from X into Y' equals by definition 'the set of ordered pairs of x and y, (x,y), such that there is a K such that (x,y) is an element of K, and K is contained in the Cartesian product of X and Y, and for all v, v is an element of X only if there is a unique u such that the ordered pair of v and u is an element of K '.

12.3. Through a function every element in X is paired with one and only one element in Y. A value of the function φ is set forth as $\varphi(x)$ and is such that $\varphi(x) = y$ where $(x,y) \in \varphi$, i.e. y is the value with which x is paired through φ. With respect to φ, X is the domain of the function, $D(\varphi)$, and Y is the image space, $I(\varphi)$, i.e. $D(\varphi) = X$ and $I(\varphi) = Y$. The range of the function, $R(\varphi)$, is that subset of Y onto which the elements of X are mapped.

13. Functional Composition

13.1. $f * g =_{Df} \{(x,h(x)) \mid g \mid X \to Y \wedge f \mid Y \to Z \wedge h \mid X \to Z \wedge h(x) = f(g(x))\}$

13.2. 'The functional composition of f and g' equals by definition 'the set of ordered pairs of x and function, h, at x, h(x), such that the function, g, defined from X into Y, and the function, f, defined from Y into Z, and h defined from X into Z, and h(x) is equal to f at g at x '.

14. Sequence

14.1. $\mathrm{seq}\ X =_{Df} \{(i,x) \mid i \in J \wedge x \in X \wedge \exists\varphi(\varphi \mid J \to X \wedge{:}\ J \sim N \wedge N = \{y \mid y = 1, 2, \ldots\} \ .\vee.\ J \sim J_n \wedge J_n = \{y \mid y = 1, 2, \ldots, n \wedge n < \infty\}\}$

14.2. 'A sequence on X' equals by definition 'the set of ordered pairs of i and x such that i is an element of J, and x is an element of X, and there is a function, φ, such that φ defined from J into X and J is equivalent to N and N is equal to the set of y such that y is equal to 1, 2, etc., or J is equivalent to J_n and J_n is equal to the set of y such that y is equal to 1, 2, etc., and n, and n is less than infinity '.

15. Power Set

15.1. $X^{2^n} =_{Df} \{A \mid A \subset X\}$

15.2. 'The power set of X' equals by definition 'the set of A such that A is contained in X '.

16. Null Set

16.1. $\emptyset =_{Df} \{\ \}$

16.2. 'The null set' equals by definition 'the set consisting of no elements '.

17. Binary Operation

17.1.
$$\ominus \mid Z \times Z \to Z =_{Df} \{((x,y),\ominus(x,y)) \mid \exists\varphi(((x,y),\ominus(x,y)) \in \varphi \land \varphi \mid X \to Y \land X = Z \times Z \land Y = Z)\}$$

17.2. 'The operation, $\ominus$, defined from the Cartesian product of Z and Z into Z' equals by definition 'the set of ordered pairs of ordered pairs of x and y, (x,y), and $\ominus$ at (x,y), $((x,y),\ominus(x,y))$, such that there is a function, φ, such that $((x,y),\ominus(x,y))$ is an element of φ, and φ defined from X into Y, and X is equal to the Cartesian product of Z and Z, and Y is equal to Z '.

17.3. The value of the binary operation, $\ominus$, will be written $x \ominus y$ and read "$\ominus$ of x and y".

18. Homomorphism

18.1.
$$\beta \mid X \to Y =_{Df} \{(x,y) \mid \exists\varphi((x,y) \in \varphi \land \varphi \mid X \to Y \land R(\varphi) = I(\varphi) \land \forall\varphi_1(\varphi_1 \mid X \to X \Rightarrow \exists\varphi_2(\varphi_2 \mid Y \to Y \land \forall(x_1,x_2)((x_1,x_2) \in \varphi_1 \Leftrightarrow (\varphi(x_1),\varphi(x_2)) \in \varphi_2))) \land \forall\ominus(\ominus \mid X \times X \to X \Rightarrow \exists\oplus(\oplus \mid Y \times Y \to Y \land \forall v_1\forall v_2(\varphi(v_1 \ominus v_2) = \varphi(v_1) \oplus \varphi(v_2)))))\}$$

18.2. 'The homomorphic mapping, β, defined from X onto Y' equals by definition 'the set of pairs of x and y, (x,y), such that there is a function, φ, such that (x,y) is an element of φ and φ defined from X into Y and the range of φ is equal to the image space of φ, and for all functions, φ_1, φ_1 defined from X into X only if there is a function, φ_2, such that φ_2 defined from Y into Y and for all ordered pairs of x_1 and x_2, (x_1,x_2), (x_1,x_2) is an element of φ_1 if and only if the ordered pair of φ_1 at x_1 and φ_2 at x_2 is an element of φ_2, and for all operations, $\ominus$, $\ominus$ defined from the Cartesian product of X and X into X only if there is an operation, $\oplus$, such that $\oplus$ defined from the Cartesian product of Y and Y into Y and for all v_1 and for all v_2, φ at $\ominus$ of v_1 and v_2 is equal to $\oplus$ of φ at v_1 and φ at v_2 '.

19. Isomorphism

19.1. $\alpha \mid X \rightarrow Y =_{Df} \{(x,y) \mid \exists\beta((x,y) \in \beta \wedge \beta \mid X \rightarrow Y \wedge \forall x_1(x_1 \in X \Rightarrow \forall x_2(x_2 \in X \wedge x_1 \neq x_2 .\Rightarrow \beta(x_1) \neq \beta(x_2))))\}$

19.2. 'The isomorphic mapping, α, defined from X onto Y' equals by definition 'the set of pairs of x and y, (x,y), such that there is a homomorphism, β, such that (x,y) is an element of β, and β defined from X onto Y, and for all x_1, x_1 is an element of X, only if for all x_2, x_2 is an element of X and x_1 is not equal to x_2 only if β at x_1 is not equal to β at x_2 '.

20. Automorphism

20.1. $\alpha' \mid X \rightarrow X =_{Df} \{(x,y) \mid \exists\alpha((x,y) \in \alpha \wedge \alpha \mid X \rightarrow Y \wedge Y = X)\}$

20.2. 'The automorphic mapping, α', defined from X onto X' equals by definition 'the set of pairs of x and y, (x,y), such that there is an isomorphism, α, such that (x,y) is an element of α, and α defined from X onto Y and Y is equal to X '.

CHAPTER II

INFORMATION THEORY

Intuitive Explication of Information

Information theory is theory which characterizes information. Information is a characterization of occurrences. An example would be biological information which is a characterization of occurences with respect to living organisms. The occurrences are characterized by means of categories. Also characterizations of occurrences sometimes are themselves made into other characterizations. An illustration would be telegraphy in which a message characterized in terms of categories of letters is made into a characterization in terms of categories of dots, dashes, and spaces. In communication, moreover, whether the characterizations are of occurrences or of other characterizations, the concern is with their transmission; such is the case, of course, in telegraphy.

'Information', however, takes on two different senses depending upon whether there are alternatives in the characterization. In the characterization, 'C_6H_6 is the formula for benzene', there are no alternatives. From a non-selective point of view the characterization is information. From a selective point of view the characterization is not information, since there are no alternatives. There is no uncertainty. In the characterization, 'cancer is either related to smoking or is not so related', there is an alternative. From a selective point of view this characterization is information. There is uncertainty, because not all occurrences can be characterized by means of one category of the two.

It should be patent from this intuitive explication of information that information theory has been extended beyond its earliest formulation. In that formulation the purpose of information theory was "to set up a quantitative measure whereby the capacities of various systems [in electrical communication] to transmit information may be compared".[1] In its more recent formulations information theory has been extended beyond quantitative measures and even beyond selective information theory within a communication context.[2]

In the development of information theory to follow, information is taken in a selective sense and within in a communication context. There is, however, an extension beyond quantitative measures. Such a development requires at least an intuitive explication of probability.

Intuitive Explication of Probability

Probability theory is mathematical theory which characterizes frequencies of occurrences with respect to classifications, i.e. sets of categories. An occurrence is said to be at a category of a classification, if it is assigned to that category. The probability that a given occurrence can be so assigned represents the ratio of the frequency of occurrences at that category to the frequency of all occurrences at every category of the classification.

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[1] R. V. L. Hartley, "Transmission of Information," Bell System Technical Journal, Vol. 7, 1928, p. 535.

[2] Donald M. McKay, "The Nomenclature of Information," Cybernetics, ed. by H. von Foerster, New York: Josiah M. Macy Junior Foundation, 1951, pp. 222-235.

Notations

1. c or any indexed c will be used for a category.
2. C or any indexed C will be used for a classification.
3. C or any indexed C will be used for a family of classifications.
4. p will be used for probability.

Development of Information Theory

1. Classifications

1.1. Simple Classification

1.1.1. $C =_{Df} \{c_i\}_{i=1}^{m}$

1.1.2. 'C' equals by definition 'set of c_i where c is indexed from 1 to m '.

1.2. Joint Classification

1.2.1. With Respect to Two Classifications

1.2.1.1. $C_{IJ} =_{Df} \{c_{ij} \mid c_{ij} = (c_i, c_j) \wedge c_i \in C_I \wedge c_j \in C_J\}$

1.2.1.2. 'C_{IJ}' equals by definition 'set of c_{ij} such that c_{ij} is equal to the ordered pair of c_i and c_j, (c_i, c_j), and c_i is an element of C_I and c_j is an element of C_J '.

1.2.1.3. C_{IJ} can be represented by an m by n matrix, where c_{ij} is in the i-th column and the j-th row.

1.2.2. With Respect to n Classifications

1.2.2.1. $C_{I_1 I_2 \ldots I_n} =_{Df} \{c_{i_1 i_2 \ldots i_n} \mid c_{i_1 i_2 \ldots i_n} = (c_{i_1}, c_{i_2}, \ldots, c_{i_n}) \wedge \bigwedge_{j=1}^{n} c_{i_j} \in C_{I_j}\}$

1.2.2.2. '$C_{I_1 I_2 \ldots I_n}$' equals by definition 'set of $c_{i_1 i_2 \ldots i_n}$ such that $c_{i_1 i_2 \ldots i_n}$ is equal to the n-tuple of c_{i_1}, c_{i_2}, etc., and c_{i_n} and the conjunction of: c_{i_j} is an element of C_{I_j}, where the variable of the conjuncts is indexed from 1 to n '.

1.2.2.3. It is difficult to represent in matrix form a joint classification of more than two classifications.

1.3. Conditional Classification of One Classification Given Another Classification

1.3.1. $C_{I|J} =_{Df} \{c_{i|j} \mid c_{i|j} = (c_i|c_j) \wedge c_i \in C_I \wedge c_j \in C_J\}$

1.3.2. '$C_{I|J}$' equals by definition 'set of $c_{i|j}$ such that $c_{i|j}$ is equal to c_i given c_j and c_i is an element of C_I and c_j is an element of C_J '.

1.3.3. $C_{I|J}$ can be represented by an m by n matrix, where $c_{i|j}$ is in the i-th column and the j-th row.

2. Classifications and Probability Distributions

2.1. Probability Distribution Defined on a Classification

2.1.1. $p \mid C \rightarrow V =_{Df} f \mid f \mid C \rightarrow V \wedge \forall c_i(c_i \in C \Rightarrow f(c_i) \geqq 0 \wedge$

$\forall c_i(c_i \in C \Rightarrow \forall c_j(c_j \in C \wedge i \neq j \;.\!\Rightarrow\!.\; c_i \cap c_j = \emptyset \wedge f(c_i \cup c_j) =$

$f(c_i) + f(c_j))) \wedge f(\emptyset) = 0 \wedge f(\bigcup_{c_i \in C} c_i) = 1$

2.1.2. 'p defined from C into the set of real numbers, V,' equals by definition 'function, f, such that f defined from C into V and for all c_i, c_i is an element of C only if f at c_i is greater than or equal to 0, and for all c_i, c_i is an element of C only if for all c_j, c_j is an element of C and i is not equal to j, only if the intersection of c_i and c_j is equal to the null set, $\emptyset$, and f at the union of c_i and c_j is equal to f at c_i plus f at c_j, and f at $\emptyset$ is equal to 0 and f at the union of c_i where c_i varies over C is equal to 1 '.

2.2. Relationship Between $p \mid C_{IJ} \rightarrow V$, $p \mid C_I \rightarrow V$, and $p \mid C_J \rightarrow V$

2.2.1. If $p \mid C_{IJ} \rightarrow V$ is given, then $p \mid C_I \rightarrow V$ can be determined.

2.2.1.1. For example, to determine $p(c_i)$,

$$p(c_i) = \sum_{j=1}^{n} p(c_i, c_j)$$

2.2.1.2. It can be shown that $\{(c,v) \mid c \in C_I \wedge v = p(c)\}$ is a probability distribution defined on C. Hence, $p \mid C_{IJ} \rightarrow V$ determines $p \mid C_I \rightarrow V$.

2.2.2. Similarly to the development under 2.2.1: If $p \mid C_{IJ} \rightarrow V$, then $p \mid C_J \rightarrow V$ can be determined.

2.3. Relationship Between $p \mid C_{IJ} \rightarrow V$ and $C_{I|J}$

2.3.1. If $p \mid C_{IJ} \rightarrow V$ is given, then conditional probabilities for each element of $C_{I|J}$ can be defined.

2.3.1.1. To determine $p(c_i | c_j)$,

$$p(c_i | c_j) = \frac{p(c_i, c_j)}{p(c_j)}$$

2.3.2. The function defined by $\{(c,v) \mid c \in C_{I|J} \wedge v = p(c)\}$ is not a probability distribution defined on $C_{I|J}$, since if the other requirements of the definition in 2.1.1 are met $p(\bigcup_{c \in C_{I|J}} c) > 1$.

2.3.3. Probability distributions can be defined for each classification of a family of conditional classifications.

2.3.3.1. $C_{I|J} =_{Df} \{C_j \mid C_j = \{(c_i|c_j) \mid c_i \in C_I\}\}_{j=1}^{n}$

2.3.3.2. '$C_{I|J}$'. equals by definition 'family of C_j such that C_j is equal to the set of c_i given c_j such that c_i is an element of C_I where C is indexed from 1 to n '.

2.3.3.3. For fixed j, it can be shown that $\{(c,v) \mid c \in C_j \wedge v = p(c)\}$ is a probability distribution defined on C_j.

2.3.3.3.1. Thus, associated with each $C_{I|J}$ is a $C_{I|J}$ with n C_j's as elements. Each C_j has a probability distribution which is derivable from C_{IJ}.

2.4. Stochastic Independence of Two Categories

2.4.1. If $p(c_i) = p(c_i|c_j)$, then c_i and c_j are said to be stochastically independent.

2.4.2. If c_i and c_j are stochastically independent, $p(c_i|c_j) = p(c_i) \cdot p(c_j)$, since $p(c_i|c_j) = \dfrac{p(c_i,c_j)}{p(c_j)}$.

3. Information Function

3.1. Suppose an occurrence in C at some c of C is under consideration, where $p \mid C \rightarrow V$. Further suppose that a number is to be assigned to an occurrence in C, then H(C)[3] indicates the uncertainty associated with that occurrence.

3.1.1. It can be seen that if H(C) is a measure of the uncertainty of an occurrence in C, then it is an equally suitable measure for the information in an occurrence in C.

3.1.2. Moreover, if $p(c_i)$ is greater than $p(c_j)$, then there is less information in an occurrence at c_i than in an occurrence at c_j, since there was greater certainty that there would be an occurrence at c_i. Thus, each c contributes more or less information (uncertainty) to an occurrence in C depending on p(c).

---

[3] H(x) is the usual notation.

3.1.2.1. The amount of information in an occurrence at c will be represented by $f[p(c)]$ and four assumptions will be made concerning $f[p(c)]$.

3.1.2.1.1. The amount of information in an occurrence at c will be a real number which depends only upon $p(c)$ and not on the probabilities of the other categories.

3.1.2.1.2. $f[p(c)]$ will be a continuous function of $p(c)$.

3.1.2.1.2.1. The basis for this assumption is that a small change in $p(c)$ should change only slightly the uncertainty of an occurrence at c.

3.1.2.1.3. If c_i and c_j are stochastically independent, then the probability that there is an occurrence at c_i and an occurrence at c_j is $p(c_i) \cdot p(c_j)$. The amount of information in such a joint occurrence will be the sum of the amount of information in each occurrence, i.e.

$$f[p(c_i) \cdot p(c_j)] = f[p(c_i)] + f[p(c_j)]$$

3.1.2.1.3.1. The basis for this assumption is the following: provided that occurrences at c_i and c_j are independent of one another, if there are 3 units of information in an occurrence at c_i and 2 units of information in an occurrence at c_j, then there should be a total of 5 units of information in their joint occurrence.

3.1.2.1.4. $f[\frac{1}{2}] = 1$

3.1.2.1.4.1. This assumption fixes the scale of the measure f and will be justified later.

3.2. Given the four assumptions under 3.1.2, the form of $f[p]$ for any probability, p, can be derived.

3.2.1. The relationship $f[p^n] = nf[p]$ holds for all real n.

3.2.1.1. If n is an integer, repeated applications of the formula in 3.1.2.1.3 yields

$$f[p^n] = nf[p]$$

3.2.1.2. To prove the statement in 3.2.1 for rational numbers, $\frac{m}{n}$, let $q = p^{m/n}$ $(q^n = p^m)$. Then by 3.2.1.1

$$f[p^{m/n}] = f[q] = \frac{1}{n}f[q^n] = \frac{1}{n}f[p^m] = \frac{m}{n}f[p]$$

3.2.1.3. To prove the statement in 3.2.1 for all real numbers, w, recall that for any w there are rational numbers arbitrarily close to w; hence by the assumption of continuity

$$f[p^w] = wf[p]$$

3.2.1.4. Assume that $(\frac{1}{2})^w = p$. By 3.2.1.3

$$f[p] = f[(\tfrac{1}{2})^w] = wf[\tfrac{1}{2}].$$

Since $p = (\frac{1}{2})^w$,

$$\log_2 p = \log_2(\tfrac{1}{2})^w = w \log_2(\tfrac{1}{2}) = - w.$$

Hence,

$$w = - \log_2 p.$$

Also by 3.1.2.1.4

$$f[p] = wf[\tfrac{1}{2}] = - \log_2 p.$$

3.2.1.4.1. Thus, we have the form of the expression for the amount of information in an occurrence at c with an a priori probability, $p(c)$.

3.3. The total information associated with an occurrence in C at some c is defined to be the weighted sum, i.e. the expected value of the information in an occurrence at each c in C:

$$H(C) =_{Df} - \sum_{i=1}^{n} p(c_i) \log_2 p(c_i) \qquad (4)$$

---

[4] Now 3.1.2.1.4 can be justified. Consider the simplest occurrence: an occurrence in C, where $C = \{c_1, c_2\}$ and $p(c_1) = p(c_2) = \frac{1}{2}$, e.g. the flipping of an unbiased coin. In this case, $H(C) = \frac{1}{2} \log_2 2 + \frac{1}{2} \log_2 2 = 1$. Thus, $H(C)$ has been defined so that the simplest occurrence has one unit of information (one bit) associated with it.

3.3.1. Since

$$-\log_2 p(c) = \log_2 \frac{1}{p(c)}$$

It is also true that

$$H(C) = \sum_{i=1}^{n} p(c_i) \log_2 \frac{1}{p(c_i)}$$

4. Information in a Joint Classification

4.1. Let C_{IJ} be given, and let the matrix

$$\begin{pmatrix} p(c_1,c_1') & \cdots & p(c_1,c_n') \\ \vdots & & \vdots \\ p(c_m,c_1') & \cdots & p(c_m,c_n') \end{pmatrix}$$

be such that $p(c_i,c_j')$ is the probability of both an occurrence at c_i and an occurrence at c_j', i.e. the matrix is the probability distribution defined on C_{IJ}.

4.1.1. The information associated with an occurrence in C_{IJ} at some c is defined similarly to the definition in 3.3.

$$H(C_{IJ}) =_{Df} -\sum_{i=1}^{m} \sum_{j=1}^{n} p(c_i,c_j') \log_2 p(c_i,c_j')$$

4.1.2. $H(C_I)$ and $H(C_J)$ can be derived from the joint probability matrix.

4.1.2.1. From probability theory if j is fixed,

$$p(c_j') = \sum_{i=1}^{m} p(c_i,c_j')$$

and if i is fixed

$$p(c_i) = \sum_{j=1}^{n} p(c_i,c_j')$$

4.1.2.2. Since the probability distributions of C_I and C_J are both obtainable from the joint probability distribution, $H(C_I)$ and $H(C_J)$ are derivable also.

$$H(C_I) = -\sum_{i=1}^{m} p(c_i) \log_2 p(c_i)$$

$$= \sum_{i=1}^{m} [(\sum_{j=1}^{n} p(c_i,c_j')) \log_2(\sum_{j=1}^{n} p(c_i,c_j'))]$$

$$H(C_J) = -\sum_{j=1}^{n} p(c_j') \log_2 p(c_j')$$

$$= -\sum_{j=1}^{n} [(\sum_{i=1}^{m} p(c_i,c_j')) \log_2(\sum_{i=1}^{m} p(c_i,c_j'))]$$

5. Information in a Conditional Classification

5.1. Let C_{IJ} and $C_{I|J}$ be given, and let the matrix

$$\begin{pmatrix} p(c_1|c_1') & \cdots & p(c_1|c_n') \\ \vdots & & \vdots \\ p(c_m|c_1') & \cdots & p(c_m|c_n') \end{pmatrix}$$

be such that $p(c_i|c_j')$ is the probability of an occurrence at c_i given c_j'.

5.1.1. The information associated with an occurrence in $C_{I|J}$ at some c is defined to be the weighted sum of the appropriate conditional probabilities.

$$H(C_I|C_J) =_{Df} \sum_{i=1}^{m} \sum_{j=1}^{n} p(c_i,c_j') \log_2 p(c_i|c_j')$$

5.1.2. $H(C_I|C_J)$ can be derived from the joint probability matrix.

5.1.2.1. From probability theory

$$p(c_i|c_j') = \frac{p(c_i,c_j')}{p(c_j)}$$

5.1.2.2. From 5.1.2.1 and 4.1.2.1

$$H(C_I|C_J) = -\sum_{i=1}^{m}\sum_{j=1}^{n} p(c_i,c_j')\ \log_2 \frac{p(c_i,c_j')}{p(c_i)}$$

$$= -\sum_{i=1}^{m}\sum_{j=1}^{n} p(c_i,c_j')\ \log_2 \frac{p(c_i,c_j')}{\sum_{j=1}^{n} p(c_i,c_j')}$$

5.2. Let $C_{I,J}$ and $C_{J|I}$ be given, and let the matrix

$$\begin{pmatrix} p(c_1'|c_1) & \cdots & p(c_n'|c_1) \\ \vdots & & \vdots \\ p(c_1'|c_m) & \cdots & p(c_n'|c_m) \end{pmatrix}$$

be such that $p(c_j'|c_i)$ is the probability of an occurrence at c_j' given c_i.

5.2.1. $H(C_J|C_I)$ can be developed as $H(C_I|C_J)$.

6. Maximum Information in a Classification

6.1. Let C be a given classification and let $\{p_i|C \rightarrow V\}_{i=1}^{n}$ be a set of probability distributions defined on C. Let $H_i(C)$ be the information associated with an occurrence at some c in C with respect to probability distribution $p_i|C \rightarrow V$.

6.1.1. $\max H(C) =_{Df} \max \{H_i(C)\}_{i=1}^{m}$

7. Shared Information Function

7.1. The following fundamental relationships hold among $H(C_I)$, $H(C_J)$, $H(C_I|C_J)$, $H(C_J|C_I)$, and $H(C_{IJ})$:

$$H(C_I) + H(C_J|C_I) = H(C_{IJ})$$

$$H(C_J) + H(C_I|C_J) = H(C_{IJ})$$

7.1.1. Figure 1 explicates the relations in 7.1.

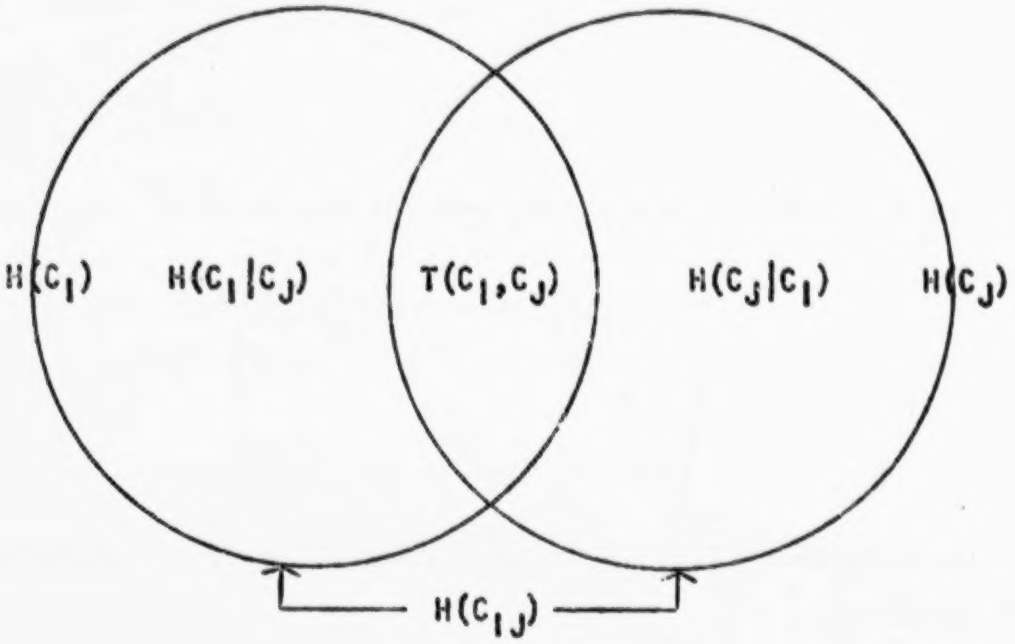

Figure 1

7.2. The information shared by C_I and C_J, $T(C_I,C_J)$, therefore, is defined as follows:

$$T(C_I,C_J) =_{Df} H(C_I) + H(C_J) - H(C_{IJ})$$

7.2.1. Since according to 7.1 $H(C_{IJ}) = H(C_I) + H(C_J|C_I)$, two alternate expressions for $T(C_I,C_J)$ are as follows:

$$T(C_I,C_J) = H(C_J) - H(C_J|C_I)$$

$$T(C_I,C_J) = H(C_I) - H(C_I|C_J)$$

7.2.2. Figure 1 also explicates the relations in 7.2.

8. Multivariate Analysis of Information Functions

8.1. To extend the analysis of information theory to the case of three or more classifications the following generalizations of the H-function and T-function are introduced:

$$H(C_{i_1 i_2 \ldots i_n}) =_{Df}$$

$$\sum_{i_1=1, i_2=1, \ldots, i_n=1}^{m_1 \quad m_2 \quad \ldots \quad m_n} p(c_{i_1}, c_{i_2}, \ldots, c_{i_n}) \log \frac{1}{p(c_{i_1}, c_{i_2}, \ldots, c_{i_n})}$$

$$\overline{T}(C_{i_1}, C_{i_2}, \ldots, C_{i_n}) =_{Df}$$

$$H(C_{i_1}) + H(C_{i_2}) + \ldots + H(C_{i_n}) - H(C_{i_1 i_2 \ldots i_n})$$

$$\overline{T}(C_{i_1 i_2 \ldots i_m}, C_{J_1 J_2 \ldots J_n}, \ldots, C_{K_1 K_2 \ldots K_q}) =_{Df}$$

$$H(C_{i_1 i_2 \ldots i_m}) + H(C_{J_1 J_2 \ldots J_n}) + \ldots + H(C_{K_1 K_2 \ldots K_q}) -$$

$$H(C_{i_1 i_2 \ldots i_m J_1 J_2 \ldots J_n K_1 K_2 \ldots K_q})$$

8.2. T-functions of different orders which measure the interaction between classifications now can be defined.

8.2.1. T-function of order 1 is defined as follows:

$$T(C_{i_1}) =_{Df} H(C_{i_1})$$

8.2.2. T-function of order 2 is defined as follows:

$$T(C_{i_1}, C_{i_2}) =_{Df} H(C_{i_1}) + H(C_{i_2}) - H(C_{i_1 i_2})$$

8.2.3. T-function of order 3 is defined as follows:

$$T(C_{I_1},C_{I_2},C_{I_3}) =_{Df} - H(C_{I_1}) - H(C_{I_2}) - H(C_{I_3}) + H(C_{I_1I_2}) + H(C_{I_1I_3}) + H(C_{I_2I_3}) - H(C_{I_1I_2I_3})$$

8.2.3. T-function of order 4 is defined as follows:

$$T(C_{I_1},C_{I_2},C_{I_3},C_{I_4}) =_{Df} H(C_{I_1}) + H(C_{I_2}) + H(C_{I_3}) + H(C_{I_4}) - H(C_{I_1I_2}) - H(C_{I_1I_3}) - H(C_{I_1I_4}) - H(C_{I_2I_3}) - H(C_{I_2I_4}) - H(C_{I_3I_4}) + H(C_{I_1I_2I_3}) + H(C_{I_1I_2I_4}) + H(C_{I_1I_3I_4}) + H(C_{I_2I_3I_4}) - H(C_{I_1I_2I_3I_4})$$

8.2.4.1. $T(C_{I_1},C_{I_2},C_{I_3},C_{I_4})$, for example, can be represented by the shaded areas in Figure 2 and is the information shared by these four classifications.

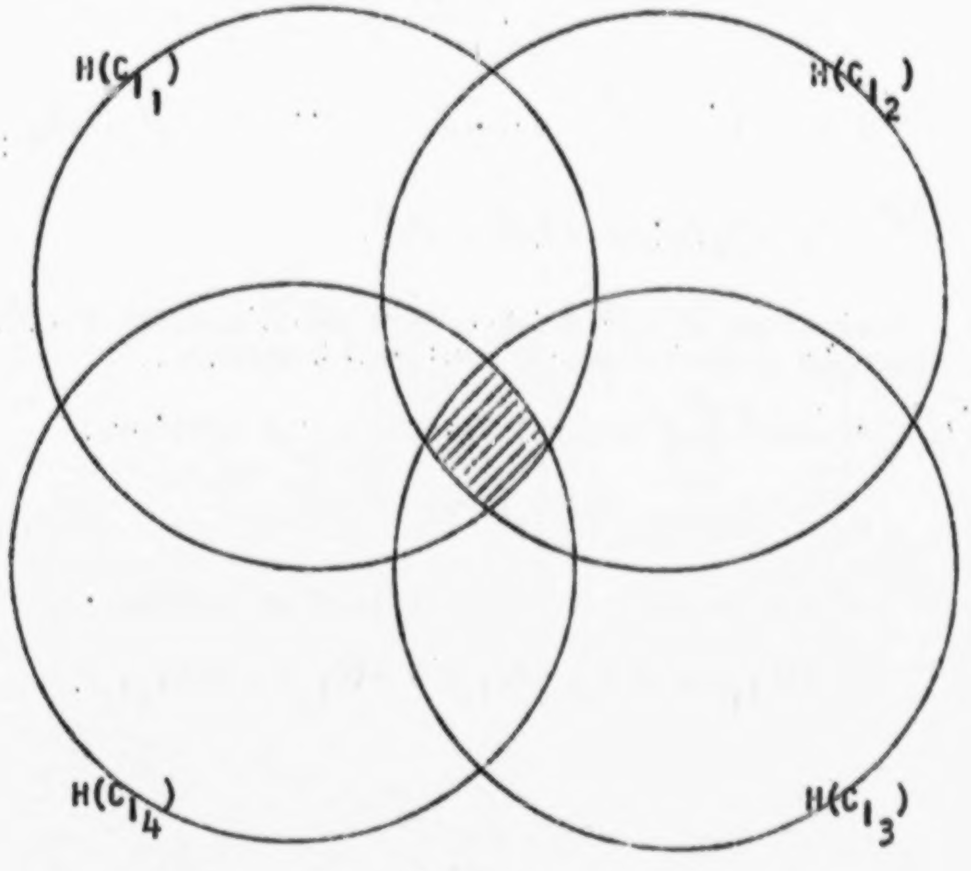

Figure 2

8.2.5. T-function of order n is defined as

$$T(C_{I_1},C_{I_2},\dots,C_{I_n}) =_{Df} (-1)^n \sum_{i=1}^{n} H(C_{I_i}) +$$

$$(-1)^{n+1} \sum_{\substack{1\leq i\leq n-1\\ 2\leq j\leq n\\ i<j}} H(C_{I_iI_j}) + \dots + (-1)\, H(C_{I_1I_2\dots I_n})$$

8.3. A B-function which measures the effect of one classification upon the relatedness of other classifications can be defined as follows:

$$B(C_I,C_{J_1J_2\dots J_n}) =_{Df} \overline{T}(C_I,C_{J_1}) + \overline{T}(C_I,C_{J_2}) + \dots + \overline{T}(C_I,C_{J_n}) -$$

$$\overline{T}(C_I,C_{J_1J_2\dots J_n})$$

8.3.1. In order to consider a multiple entry in the place of C_I in the B-function, analysis would have to consider all possible relations of these entries to those on the right of the comma. Thus, for the two-entry case, the following would obtain:

$$B(C_{I_1I_2},C_{J_1J_2\dots J_n}) =_{Df} \overline{T}(C_{I_1},C_{J_1}) + \overline{T}(C_{I_1},C_{J_2}) + \dots +$$

$$\overline{T}(C_{I_1},C_{J_n}) - \overline{T}(C_{I_1},C_{J_1J_2\dots J_n}) + \overline{T}(C_{I_2},C_{J_1}) + \overline{T}(C_{I_2},C_{J_2}) +$$

$$\dots + \overline{T}(C_{I_2},C_{J_n}) - \overline{T}(C_{I_2},C_{J_1J_2\dots J_n}) + \overline{T}(C_{I_1I_2},C_{J_1}) +$$

$$\overline{T}(C_{I_1I_2},C_{J_2}) + \dots + \overline{T}(C_{I_1I_2},C_{J_n}) - \overline{T}(C_{I_1I_2},C_{J_1J_2\dots J_n})$$

8.3.1.1. It is clear that the number of terms increase rapidly as the number of left entries increase. The feasibility for using the B-function for anything over the first or second entry function, therefore, is questionable.

CHAPTER III

GRAPH THEORY

Intuitive Explication of Digraph

Digraph theory is mathematical theory which characterizes between pairs of points lines which can be directed. Figures can be utilized to explicate intuitively a digraph, as in Figure 1.

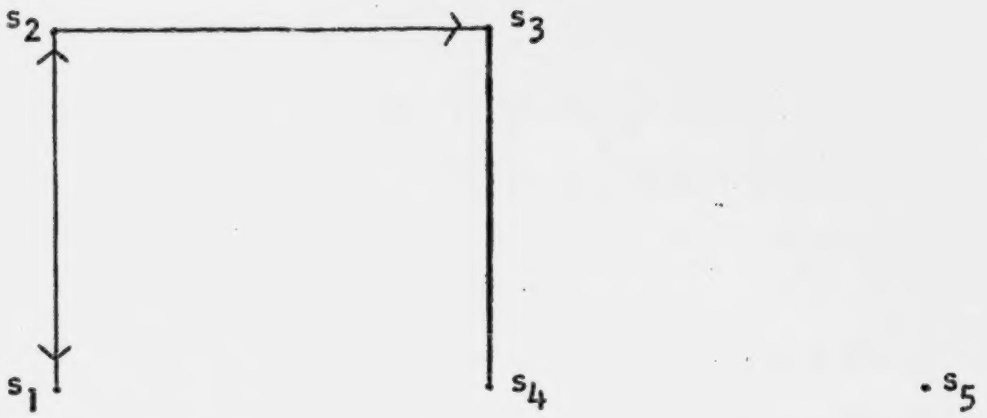

Figure 1

Figure 1 was constructed from points--s_1, s_2, s_3, s_4, s_5--and lines, some of which are arrows. In the figure the pairs of points which can be considered are as follows: s_1s_2, s_1s_3, s_1s_4, s_1s_5, s_2s_3, s_2s_4, s_2s_5, s_3s_4, s_3s_5, and s_4s_5. The following pairs do not have lines between them: s_1s_5, s_2s_5, s_3s_5, and s_4s_5. s_5, thus, has no connection with the other points. In the case of s_1s_2, s_1s_2, and s_2s_3 the lines are arrows and so establish directed connections between the points of each pair. In s_1s_2 and s_2s_3 the directed connections are direct, since there is only one arrow from s_1 to s_2 and from s_2 to s_3. In s_1s_3 the directed connection is indirect, since there is more than one arrow from s_1 to s_3. Between s_3 and s_4 there is a line that is not an arrow. Apparently there is no directed connection. A directed line between s_2 and s_4, however, will be assumed, i.e. there are assumed arrows from s_3 or s_4 or both. The result of such an assumption is the treatment of graph theory within the context of digraph[1] theory. Interchangeable usage of the terms, 'graph theory' and 'digraph theory', therefore, is permitted.

Notations

1. G or any indexed G will be used for a digraph.
2. s or any indexed s will be used for a point.
3. S or any indexed S will be used for a set of points.
4. (x,y), where x and y are points, will be used for a line directed from x to y.

Development of Digraph Theory

1. Finite Digraph

   1.1. $G =_{Df} (S,R) \mid S = \{s_i\}_{i=1}^{n} \wedge R \subset S \times S \wedge \forall s_i (s_i \in S \Rightarrow (s_i,s_i) \notin R)$

   1.2. 'G' equals by definition 'the ordered pair of S and R such that S is a set of s_i where s is indexed from 1 to n, and R is contained in the Cartesian product of S and S, and for all s_i, s_i is an element of S only if (s_i,s_i) is not an element of R '.

2. Gamma Function of a Point

   2.1. Since every binary relation is a set of ordered pairs, the relations of a digraph can be explicated as a mapping (not necessarily single valued) from the set of points of the digraph into itself.

---

[1] 'Di-' indicates that the graphs consist of directed lines.

2.1.1. $\Gamma(s) =_{Df} \{s' \mid (s,s') \in R\}$ (2)

2.1.2. 'Gamma, Γ, at s' equals by definition 'set of s' such that (s,s') is an element of R '.

2.2. In Figure 2, $\Gamma(s_2)$ is the set consisting of the points s_1 and s_3, $\{s_1,s_3\}$[3], since directed lines, (s_2,s_3) and (s_2,s_1) are in the graph.

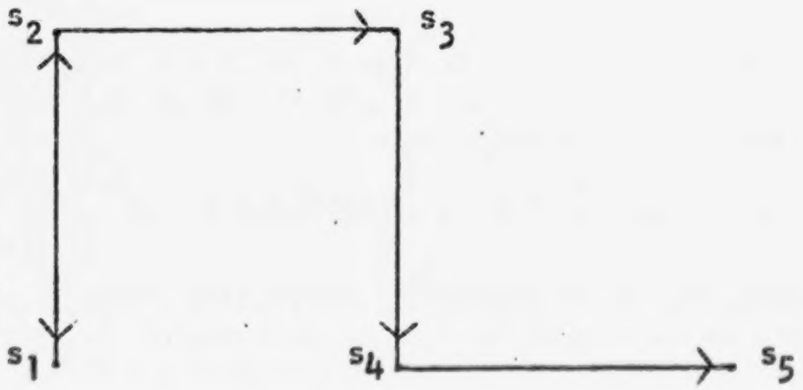

Figure 2

2.3. Stated less formally, $\Gamma(s)$ consists of all s' such that there is a directed line from s to s'.

2.3.1. Γ is said to map s into $\Gamma(s)$.

---

[2] Because the gamma function and all functions derived from it are characterizations of the relation of a digraph as a mapping, a digraph with specified set and relation will be assumed.

[3] This is not to be confused with the ordered pair of s_1 and s_3, (s_1,s_3).

3. Inverse Gamma Function of a Point

3.1. It is possible to characterize analytically the set of all points which are connected by a single line directed to a given point.

3.1.1. $\bar{\Gamma}(s) =_{Df} \{s' \mid (s',s) \in R\}$

3.1.2. 'Gamma inverse, $\bar{\Gamma}$, at s' equals by definition 'set of s' such that (s',s) is an element of R '.

3.2. In Figure 2, $\bar{\Gamma}(s_2)$ is the set consisting of s_1, $\{s_1\}$, since (s_1,s_2) is in the digraph.

4. Gamma Function of a Set

4.1. By generalizing the gamma function of a point, one can define the set of all points which are joined to a given set by a directed line from that set.

4.1.1. $\Gamma(S) =_{Df} \{s' \mid \exists s(s \in S \wedge (s,s') \in R \wedge s' \notin S)\}$

4.1.2. 'Gamma, Γ, at S' equals by definition 'set of s' such that there is an s such that s is an element of S and (s,s') is an element of R and s' is not an element of S '.

4.2. In Figure 3, $\Gamma(\{s_2,s_3,s_5\}) = \{s_1,s_6\}$, since $s_3 \in S \wedge (s_3,s_1) \in R$

$\wedge\ s_1 \notin S$, and $s_5 \in S \wedge (s_5,s_6) \in R \wedge s_6 \notin S$.

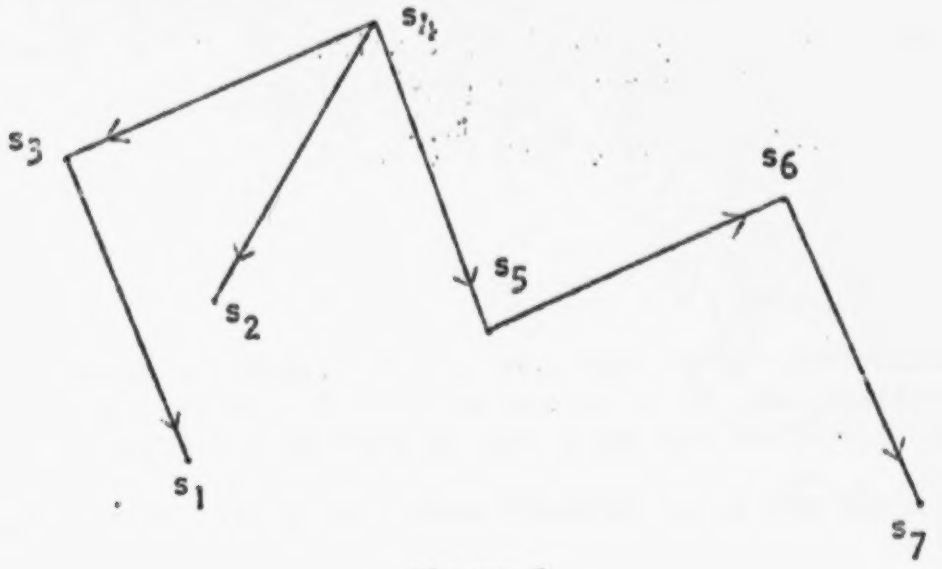

Figure 3

5. Transitive Closure of a Point

5.1. There is a directed path from one point to another, if there is a sequence of directed lines such that one can trace along this sequence by following the direction of the arrows.

5.1.1. There is a directed path from s_4 to s_7 in Figure 3, since there is a sequence of directed lines such that one can trace from s_4 to s_5 to s_6 to s_7 by following the direction of the arrows.

5.2. The definition in 4.1.1 makes it possible to characterize analytically the occurrence of a directed path between two points.

5.2.1. In Figure 3 the set of all points to which there is a directed path from point s_4 can be specified.

5.2.1.1. $\Gamma(s_4) = \{s_2, s_3, s_5\}$

5.2.1.2. $\Gamma^2(s_4) = \Gamma(\Gamma(s_4)) = \Gamma(\{s_2, s_3, s_5\}) = (\{s_1, s_6\})$

5.2.1.3. $\Gamma^3(s_4) = \Gamma(\Gamma^2(s_4)) = \Gamma\{s_1, s_6\} = \{s_7\}$

5.2.2. Recalling the characterization of directed path in 5.1, it can be seen that the points contained in $\Gamma(s_4)$, $\Gamma^2(s_4)$, and $\Gamma^3(s_4)$ are all the points to which there is a directed path from s_4. Hence using the set theoretic operation of union, $\Gamma(s_4) \cup \Gamma^2(s_4) \cup \Gamma^3(s_4)$ is the set to be specified in 5.2.1.

5.3. The example under 5.2 clarifies the following inductive definition of gamma i of a point where 3.1.1 is the basic step and the following is the inductive step.

5.3.1. $\Gamma^i(s) =_{Df} \Gamma(\Gamma^{i-1}(s))$

5.3.2. 'Gamma i, Γ^i, at s' equals by definition 'gamma, Γ, at gamma i minus 1, Γ^{i-1}, at s '.

5.4. Following the example under 5.2 the set of all points to which there is a directed path from a point, the transitive closure of a point, is defined as the union of the gamma i's of the point.

5.4.1. $\Gamma_o(s) =_{Df} \bigcup_{i=1}^{\infty} \Gamma^i(s)$ (4)

5.4.2. 'The transitive closure, Γ_o, at s' equals by definition 'the union of gamma i, Γ^i, at s where Γ is indexed from 1 to infinity, ∞ '.

5.5. Hence the statement, '$s' \in \Gamma_o(s)$', is read "s′ is an element of the transitive closure of s," but can be interpreted as follows: there is a directed path from s to s′.

6. Inverse Transitive Closure of a Point

6.1. The inverse gamma function of a set characterizes the set of points from which there is a directed line to the set.

6.1.1. $\overline{\Gamma}(S) =_{Df} \{s' \mid \exists s(s \in S \wedge (s',s) \in R \wedge s' \notin S)\}$

6.1.2. 'Gamma inverse, $\overline{\Gamma}$, at S' equals by definition 'set of s′ such that there is an s such that s is an element of S and (s′,s) is an element of R and s′ is not an element of S '.

6.2. The set of all points from which there is a directed path to a given point can be defined as follows.

6.2.1. $\overline{\Gamma}_o(s) =_{Df} \bigcup_{i=1}^{\infty} \overline{\Gamma}^i(s)$

6.2.2. 'The inverse transitive closure, $\overline{\Gamma}_o$, at s' equals by definition 'the union of gamma inverse i, $\overline{\Gamma}^i$, at s where $\overline{\Gamma}$ is indexed from 1 to infinity, ∞ '.

6.3. Hence the statement '$s' \in \overline{\Gamma}_o(s)$' is read "s′ is an element of the inverse transitive closure of s," but can be interpreted as follows: there is a directed path from s′ to s.

---

[4] $\Gamma^1(s)$ is defined as $\Gamma(s)$.

7. Reciprocation of a Digraph

7.1. The reciprocation of a digraph is a digraph in which each one-way relation is replaced by a two-way relation.

7.1.1. $*G =_{Df} (S',R') \mid S' = S \land \forall(s_1,s_2)((s_1,s_2) \in R \Rightarrow.$

$(s_1,s_2) \in R' \land (s_2,s_1) \in R')$

7.1.2. 'Reciprocation, *, of G' equals by definition '(S',R') such that S' is equal to S, and for all (s_1,s_2), (s_1,s_2) is an element of R, only if (s_1,s_2) is an element of R' and (s_2,s_1) is an element of R' '.

8. Semi-transitive Closure of a Point

8.1. There is a semi-path from one point to another, if there is a sequence of directed lines such that one can trace along this sequence either by following or ignoring the direction of the arrows.

8.1.1. There is a semi-path from s_2 to s_5 in Figure 3, since there is a sequence of directed lines such that one can trace against the indicated direction from s_2 to s_4 and with the indicated direction from s_4 to s_5.

8.2. The semi-transitive closure of a point is defined so that the notion of a semi-path from a point may be made precise.

8.2.1. $\Sigma_o(s) =_{Df} \{s' \mid s' \in \Gamma_o^{*G}(s)\}$ (5)

8.2.2. 'Semi-transitive closure, Σ_o, at s' equals by definition 'set of s' such that s' is an element of the transitive closure, Γ_o, at s with respect to the reciprocation, *, of G '.

8.3. The statement '$s' \in \Sigma_o(s)$' is read "s' is an element of the semi-transitive closure of s," but can be interpreted as follows: there is a semi-path from s to s'.

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[5] Whenever a function is taken with respect to a digraph which differs from the understood digraph, that function will be appropriately indexed.

9. Affect Function of a Digraph

9.1. The set of all directed lines which form directed paths from a given point to points in the digraph is developed below.

9.1.1. Recall the example in 5.2.1.

9.1.1.1. $\Gamma(s_4) = \{s_2, s_3, s_5\}$, since (s_4, s_2), (s_4, s_3), and (s_4, s_5) were in the digraph.

9.1.1.1.1. Denote the set $\{(s_4, s_2), (s_4, s_3), (s_4, s_5)\}$ by $\Delta(s_4)$. This is precisely the set of all lines directed from s_4.

9.1.1.2. $\Gamma^2(s_4) = \Gamma(\Gamma(s_4)) = \Gamma(\{s_2, s_3, s_5\}) = \{s_1, s_6\}$, since (s_3, s_1) and (s_5, s_6) were in the digraph.

9.1.1.2.1. Denote the set $\{(s_3, s_1), (s_5, s_6)\}$ by $\Delta^2(s_4)$. This is the set of all directed lines one step removed from s_4.

9.1.1.3. Likewise $\Delta^3(s_4) = \{(s_6, s_7)\}$.

9.1.1.4. If $\Delta_o(s_4)$ now denotes $\Delta(s_4) \cup \Delta^2(s_4) \cup \Delta^3(s_4)$, it is seen that $\Delta_o(s_4)$ is the set of all directed lines which form directed paths from s_4 to points in the digraph.

9.1.2. The example under 9.1.1 illustrates the definitions given below.

9.1.2.1. $\Delta(S) =_{Df} \{(s', s) \mid \exists s'(s' \in S \wedge (s', s) \in R)\}$

9.1.2.2. 'Delta, Δ, at S' equals by definition 'set of (s', s) such that there is an s' such that s' is an element of S and (s', s) is an element of R '.

9.1.2.3. $\Delta^i(S) =_{Df} \{(s', s) \mid (s', s) \in R \wedge \exists s''((s'', s') \in \Delta^{i-1}(S))\}$

9.1.2.4. 'Delta i, Δ^i, at S' equals by definition 'set of (s', s) such that (s', s) is an element of R, and there is an s'' such that (s'', s') is an element of delta i minus 1, Δ^{i-1}, at S '.

9.1.2.5. $\Delta_o(S) =_{Df} \bigcup_{i=1}^{\infty} \Delta^i(S)$ (6)

9.1.2.6. 'Affect function, Δ_o, at S' equals by definition 'the union of delta i, Δ^i, at S where Δ is indexed from 1 to infinity, ∞ '.

10. Distance Between Two Points

10.1. The distance between two points of a digraph is the shortest directed path from one point to another.

10.1.1. $d(s_1,s_2) =_{Df} i \mid s_2 \in \Gamma^i(s_1) \wedge \forall j(s_2 \in \Gamma^j(s_1) \Rightarrow i \leqq j)$

10.1.2. 'Distance, d, at (s_1,s_2)' equals by definition 'i such that s_2 is an element of gamma i, Γ^i, at s_1, and for all j, s_2 is an element of gamma j, Γ^j, at s_1 only if i is less than or equal to j '.

11. Diameter of a Digraph

11.1. The diameter of a digraph is the maximum distance between any two points of the digraph.

11.1.1. $D_G =_{Df} \max_{\substack{s \in S \\ s' \in S}} d(s,s')$

11.1.2. 'Diameter, D, of a G' equals by definition 'maximum distance at (s,s'), max $d(s,s')$, where s varies over S and s' varies over S '.

12. Types of Digraphs

12.1. A complete digraph is a digraph containing all possible directed lines.

12.1.1. $G_C =_{Df} G \mid \forall s_1(s_1 \in S \Rightarrow \forall s_2(s_2 \in S \Rightarrow. (s_1,s_2) \in R \wedge$

$(s_2,s_1) \in R))$

12.1.2. 'G_C' equals by definition 'G such that for all s_1, s_1 is an element of S, only if for all s_2, s_2 is an element of S

---

[6] $\Delta^1(S)$ is defined as $\Delta(S)$.

only if (s_1,s_2) is an element of R and (s_2,s_1) is an element of R '.

12.1.3. Figure 4 is a complete digraph of five points.

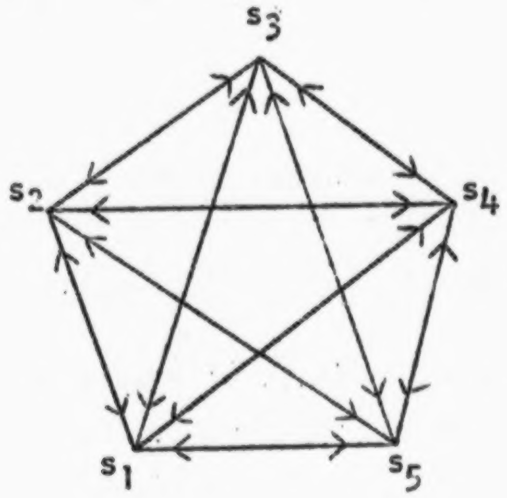

Figure 4

12.2. For any two points in a strong digraph, there is a directed path from the first point to the second point and a directed path from the second point to the first point.

12.2.1. $G_S =_{Df} G \mid \forall s_1(s_1 \in S \Rightarrow \forall s_2(s_2 \in S \Rightarrow. s_1 \in \Gamma_o(s_2) \wedge s_2 \in \Gamma_o(s_1)))$

12.2.2. 'G_S' equals by definition 'G such that for all s_1, s_1 is an element of S, only if for all s_2, s_2 is an element of S only if s_1 is an element of the transitive closure, Γ_o, at s_2 and s_2 is an element of the transitive closure, Γ_o, at s_1 '.

12.2.3. Both Figures 4 and 5 are strong digraphs.

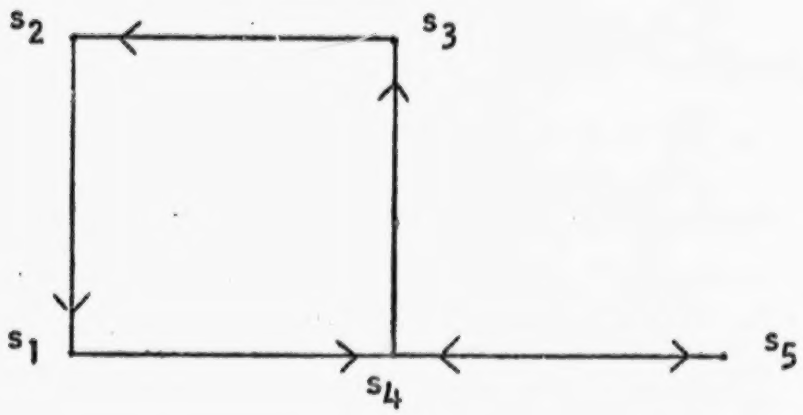

Figure 5

12.3. For any two points in a unilateral digraph, there is either a directed path from the first point to the second point or a directed path from the second point to the first point.

12.3.1. $G_U =_{Df} G \mid \forall s_1(s_1 \in S \Rightarrow \forall s_2(s_2 \in S \Rightarrow. s_1 \in \Gamma_o(s_2) \vee s_2 \in \Gamma_o(s_1)))$

12.3.2. 'G_U' equals by definition 'G such that for all s_1, s_1 is an element of S, only if for all s_2, s_2 is an element of S only if either s_1 is an element of the transitive closure, Γ_o, at s_2 or s_2 is an element of the transitive closure, Γ_o, at s_1 '.

12.3.3. Figures 4 and 5 are unilateral digraphs which are strong. Figure 6 is a unilateral digraph which is not strong, since there is not a directed path from s_1 to s_2.

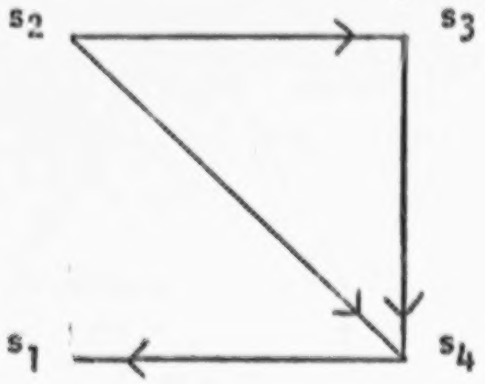

Figure 6

12.4. In a weak digraph there is a semi-path connecting every two points.[7]

12.4.1. $G_W =_{Df} G \mid \forall s_1(s_1 \in S \Rightarrow \forall s_2(s_2 \in S \Rightarrow s_1 \in \Sigma_o(s_2)))$

12.4.2. 'G_W' equals by definition 'G such that for all s_1, s_1 is an element of S, only if for all s_2, s_2 is an element of S only if s_1 is an element of the semi-transitive closure, Σ_o, at s_2 '.

12.5. A disconnected digraph is one whose points can be divided into two sets such that there is no connection between the two sets.

12.5.1. $G_D =_{Df} G \mid \exists S_1 \exists S_2(S_1 \cup S_2 = S \wedge S_1 \cap S_2 = \emptyset \wedge \forall s_1(s_1 \in S_1 \Rightarrow$

$\forall s_2(s_2 \in S_2 \Rightarrow s_1 \notin \Sigma_o(s_2))))$

12.5.2. 'G_D' equals by definition 'G such that there is an S_1 such that there is an S_2 such that the union of S_1 and S_2 is equal to S, and the intersection of S_1 and S_2 is equal to the null set, $\emptyset$, and for all s_1, s_1 is an element of S_1 only if for all s_2, s_2 is an element of S_2 only if s_1 is not an element of the semi-transitive closure, Σ_o, of s_2 '.

12.5.3. Figure 7 is a disconnected digraph.

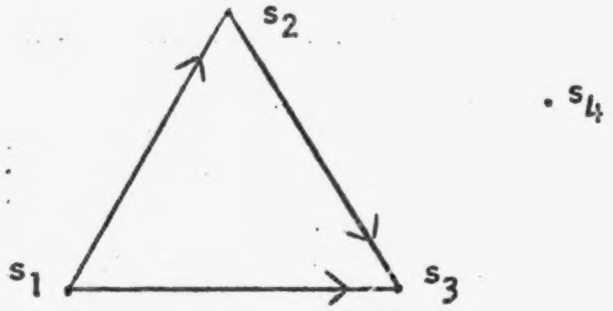

Figure 7

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[7] Weak digraphs are also called 'connected digraphs'.

12.6. These five categories of digraphs are not mutually exclusive; in fact every completely connected digraph is strong, every strong digraph is unilateral, and every unilateral digraph is weak.

12.6.1. In Figure 8 the relationships among the five categories are illustrated by the use of a diagram.

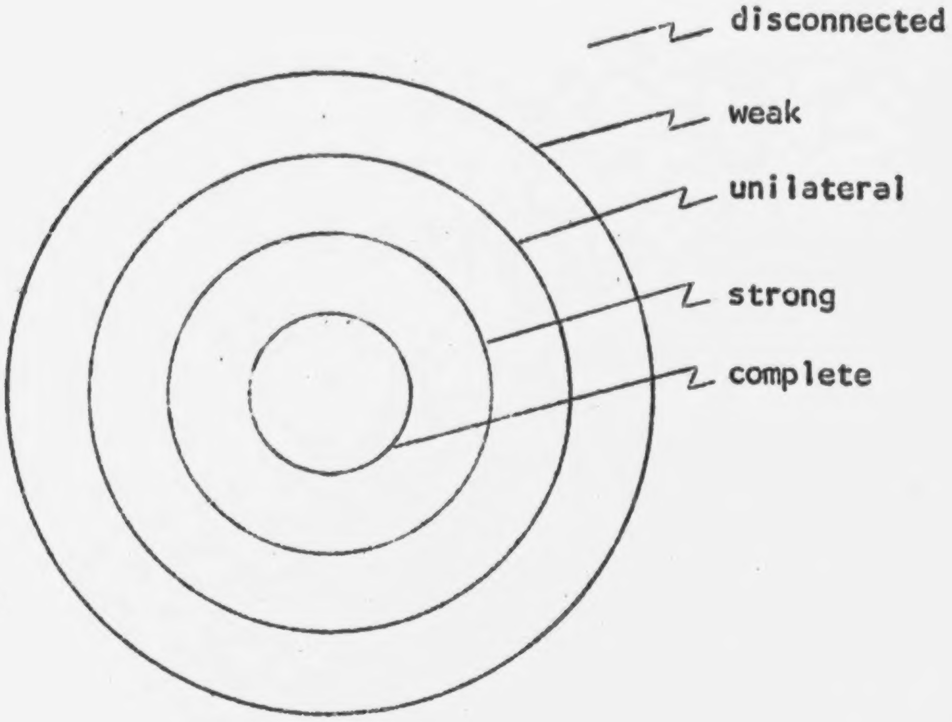

Figure 8

13. Mutually Exclusive Categories of Digraphs

13.1. It can be seen from Figure 8 that the following five sets of digraphs are mutually exclusive: 1) disconnected digraphs, 2) digraphs which are weak and not unilateral, 3) digraphs which are unilateral and not strong, 4) digraphs which are strong and not complete and, 5) complete digraphs.[8] Definitions of 1 and 5 are stated in 12.5 and 12.1 respectively. Definitions of 2, 3, and 4 are stated on the following page.

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[8] This statement is a theorem, but the proof, which involves rather straight-forward manipulation of the definitions, has been omitted due to its length.

13.1.1. $G_{SW} =_{Df} G \mid G \in \{G_W\} \wedge G \notin \{G_U\}$ (9)

13.1.2. 'Strictly weak digraph' equals by definition 'G such that G is an element of the set of G_W and G is not an element of the set of G_U '.

13.1.3. $G_{SU} =_{Df} G \mid G \in \{G_U\} \wedge G \notin \{G_S\}$

13.1.4. 'Strictly unilateral digraph' equals by definition 'G such that G is an element of the set of G_U and G is not an element of the set of G_S '.

13.1.5. $G_{SS} =_{Df} G \mid G \in \{G_S\} \wedge G \notin \{G_C\}$

13.1.6. 'Strictly strong digraph' equals by definition 'G such that G is an element of the set of G_S and G is not an element of the set of G_C '.

14. Digraph with Relations Removed

14.1. Starting with a digraph $G = (S,R)$, the digraph resulting from removing a set of directed lines, R_1, may be represented as follows: $(S,R-R_1)$

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[9] $\{G_T\}$ will be used for the set of all digraphs of type, T. The complete set theoretic characterization of this set is $\{G \mid \exists G_T(G = G_T)\}$.

CHAPTER IV

THE SIGGS THEORY MODEL

Nature of the Model

The SIGGS Theory Model consists of a group of related terms. The terms are related so that some are primitive or undefined and the others are defined. Primitive terms are required to prevent circularity. Moreover, all the defined terms are defined through primitive terms or defined terms which already were defined by means of primitive terms. Since the terms are characterizations with respect to a system in general and not with respect to only one kind of system, e.g. a biological one, the theory model can be said to be a group of related characterizations about a general system.

'SIGGS' indicates that the characterizations were developed from set theory (S), information theory (I), graph theory (G), and general systems theory (GS). Statemental and predicate calculi as well as algebra also were employed in the development. In Appendix I some indication of how predicate calculus enters into the model is presented.

Because set theory, information theory, and graph theory were utilized, advantages were gained. Development of characterizations about a general system beyond those already developed became possible. Logico-mathematical ideographs became available to give greater precision to the theory model.

Presentation of the Model

The terms are presented as follows

1. citation of term which takes the form, n. ..., ___

   where 'n' stands for a number which indicates order of presentation
   '...' stands for a term
   '___' stands for a symbol for the respective term

2. definition of terms, unless term is primitive, which takes the forms,

   2.1. natural language definition which takes the form, n.1. ... is ----.

   where '.1' stands for a natural language definition
   '...' stands for a definiendum
   '---' stands for the respective definiens

   2.2. logico-mathematical definition which takes the form, n.2. ... $=_{Df}$ ---

   where '.2' stands for a logico-mathematical definition
   '$=_{Df}$' stands for equals by definition

   2.2.1. readoff of the logico-mathematical definition, which translates the logico-mathematical symbols without cross referencing the syntactical ones, which readoff is indicated by '.3'

   2.2.1.1. cross referencing of syntactical logico-mathematical symbols to verbal ones which is presented in Appendix II.

2.2.1. and 2.2.1.1 are included, because of the difficulties attendant to the use of logico-mathematical ideographs for those not versed in such symbolism.

1. universe of discourse, $\mathcal{U}$

2. component, s

3. group, S

   3.1. A group is at least two components that form a unit within the universe of discourse.

   3.2. $S =_{Df} \{s_i \mid 1 \leqq i \wedge i \leqq n \wedge n \geqq 2\}$

   3.3. 'Group', 'S', equals by definition 'set of components, s_i, such that 1 is less than or equal to i and i is less than or equal to n and n is greater than or equal to 2 '.

4. characterization, CH

5. information, I

   5.1. Information is characterization of occurrences.

   5.2. $I =_{Df} CH \mid CH = \{c \mid \text{℩}p((c,v) \in p)\}$

   5.3. 'Information', 'I', equals by definition 'characterization, CH, such that CH is equal to a set of categories, c, such that that probability distribution, p, such that the pair of c and the real number, v, (c,v), is an element of p '.

5-1. selective information, I_S

   5-1.1. Selective information is information which has alternatives.

5-1.2. $I_S =_{Df} I \mid \exists c((c,v) \in p \wedge 0 < v \wedge v < 1)$ (1)

5-1.3. 'Selective information', 'I_S', equals by definition 'information, I, such that there is a category, c, such that the pair of c and the real number, v, (c,v), is an element of a probability distribution, p, and 0 is less than v and v is less than 1 '.

5-1-1. nonconditional selective information, I_S^N

5-1-1.1. Nonconditional selective information is selective information which does not depend on other selective information.

5-1-1.2. $I_S^N =_{Df} I_S \mid \exists n(n \geqq 1 \wedge I = C_{I_1 I_2 \ldots I_n})$ (1)

5-1-1.3. 'Nonconditional selective information', 'I_S^N', equals by definition 'selective information, I_S, such that there is an integer, n, such that n is greater than or equal to 1, and I is equal to the joint classification with respect to n classifications, $C_{I_1 I_2 \ldots I_n}$ '.

5-1-2. conditional selective information, I_S^C

5-1-2.1. Conditional selective information is selective information which depends on other selective information.

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[1] The condition is a conjunct to the defining condition of the term before '|'.

5-1-2.2. $I_S^C =_{Df} I_S \mid I \in C_{I|J}$ (1)

5-1-2.3. 'Conditional selective information', 'I_S^C', equals by definition 'selective information, I_S, such that I is an element of the family of conditional classifications, $C_{I|J}$'.

6. transmission of selective information, $\vec{I}(I_{S_1}, I_{S_2}, \ldots, I_{S_i}, \ldots, I_{S_n})$

6.1. Transmission of selective information is a flow of selective information.

6.2.
$$\vec{I}(I_{S_1}, I_{S_2}, \ldots, I_{S_i}, \ldots, I_{S_n}) =_{Df} \underset{t_i<t_{i+1}}{T}(I_{S_1}(t_1), I_{S_2}(t_2), \ldots, I_{S_i}(t_i), \ldots, I_{S_n}(t_n))$$

6.3. 'Transmission of selective information between selective information where I_S is indexed from 1 to n', '$\vec{I}(I_{S_1}, I_{S_2}, \ldots, I_{S_i}, \ldots, I_{S_n})$', equals by definition 'shared information, T, between selective information, I_S, at time, t, where I_S and t are indexed from 1 to n, and t_i precedes t_{i+1}'.

7. affect relation, R_A

7.1. An affect relation is a connection of one or more components to one or more other components.

7.2. $R_A =_{Df} R \mid R \subset S \times S \wedge R \neq \emptyset \wedge \forall(s_i,s_j)((s_i,s_j) \in R \Rightarrow.$

$s_i \in \Sigma_o(s_j) \wedge s_i \neq s_j)$

7.3. 'Affect relation', 'R_A', equals by definition 'relation, R, such that R is contained in the Cartesian product of a group, S, and S, and R is not equal to the null set, $\emptyset$, and for all pairs of component, s_i, and component, s_j, (s_i,s_j), (s_i,s_j) is an element of R only if s_i is an element of the semi-transitive closure, Σ_o, at s_j and s_i is not equal to s_j '.

7-1. directed affect relation, R_{DA}

7-1.1. A directed affect relation is an affect relation in which one or more components have a channel to one or more other components.

7-1.2. $R_{DA} =_{Df} R \mid R \subset S \times S \wedge R \neq \emptyset \wedge \forall(s_i,s_j)((s_i,s_j) \in R \Rightarrow.$

$s_j \in \Gamma_o(s_i) \wedge s_i \neq s_j)$

7-1.3. 'Directed affect relation', 'R_{DA}', equals by definition 'relation, R, such that R is contained in the Cartesian product of a group, S, and S, and R is not equal to the null set, $\emptyset$, and for all pairs of component, s_i, and component, s_j, (s_i,s_j), (s_i,s_j) is an element of R only if s_j is an element of the transitive closure, Γ_o, at s_i and s_i is not equal to s_j '.

7-1-1. direct directed affect relation, R^D_{DA}

7-1-1.1. A direct directed affect relation is a directed affect relation in which the channel is through no other components.

7-1-1.2. $R^{D}_{DA} =_{Df} R_{DA} \mid s_j \in \Gamma^1(s_i)$ (2)

7-1-1.3. 'Direct directed affect relation', 'R^{D}_{DA}', equals by definition 'directed affect relation, R_{DA}, such that component, s_j, is an element of gamma 1, Γ^1, at component, s_i '.

7-1-2. Indirect directed affect relation, R^{I}_{DA}

7-1-2.1. An indirect directed affect relation is a directed affect relation in which the channel is through other components.

7-1-2.2. $R^{I}_{DA} =_{Df} R_{DA} \mid s_j \notin \Gamma^1(s_i) \wedge s_j \in \Gamma_o(s_i)$ (2)

7-1-2.3. 'Indirect directed affect relation', 'R^{I}_{DA}', equals by definition 'directed affect relation, R_{DA}, such that component, s_j, is not an element of gamma 1, Γ^1, at component, s_i, and s_j is an element of the transitive closure, Γ_o, at component, s_i '.

8. system, $\overline{S}$

8.1. A system is a group with at least one affect relation which has information.

---

[2] The condition is a conjunct to the consequent in 7-1.2.

8.2.

$$\begin{gathered}\overline{S} =_{Df} S \mid \exists\mathcal{R}_A(\mathcal{R}_A \neq \emptyset \wedge \forall R_A(R_A \in \mathcal{R}_A \Rightarrow. R_A \subset S \times S \wedge \\ \exists\mathcal{I}(\mathcal{I} \neq \emptyset \wedge \forall I(I \in \mathcal{I} \Rightarrow. I \sim R_A \mathbin{\overline{\nabla}} \exists\mathcal{R}(\mathcal{R} \subset R_A^{2^n} \wedge I \sim \mathcal{R}) \nabla \\ \exists S'(S' \subset S \wedge I \sim S'))))) \end{gathered}$$

8.3. 'System', '$\overline{S}$' equals by definition 'group, S, such that there is a family of affect relations, $\mathcal{R}_A$, such that $\mathcal{R}_A$ is not equal to the null set, $\emptyset$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}_A$, only if R_A is contained in the Cartesian product of S and S, and there is a family of informations, $\mathcal{I}$, such that $\mathcal{I}$ is not equal to $\emptyset$, and for all information, I, I is an element of $\mathcal{I}$ only if either I is equivalent to R_A or there is a family of relations, $\mathcal{R}$, such that $\mathcal{R}$ is contained in the power set of R_A and I is equivalent to $\mathcal{R}$ or there is a group, S', such that S' is contained in S and I is equivalent to S' and I is not equivalent to any combination thereof '.

9. negasystem, $\overline{\not{S}}$

9.1. A negasystem is the components not taken to be in a system.

9.2. $\overline{\not{S}} =_{Df} C_{\mathcal{U}}S \mid C_{\mathcal{U}}S \neq \emptyset$

9.3. 'Negasystem', '$\overline{\not{S}}$', equals by definition 'complement of group, S, with respect to the universe of discourse, $\mathcal{U}$, such that the complement of S with respect to $\mathcal{U}$ is not equal to the null set, $\emptyset$ '.

10. condition, $\underline{F}$

11. system state, $ST_{\overline{S}}$

11.1. A system state is a system's conditions at a given time.

11.2. $ST_{\overline{S}} =_{Df} \{\underline{F} \mid \iota\overline{S}(\iota t(\underline{F}(\overline{S}(t))))\}$

11.3. 'System state', '$ST_{\overline{S}}$', equals by definition 'set of conditions, $\underline{F}$, such that that system, $\overline{S}$, such that that time, t, such that $\underline{F}$ at $\overline{S}$ at t '.

12. negasystem state, $ST_{\overline{\not S}}$

12.1. A negasystem state is a negasystem's conditions at a given time.

12.2. $ST_{\overline{\not S}} =_{Df} \{\underline{F} \mid \iota\overline{\not S}(\iota t(\underline{F}(\overline{\not S}(t))))\}$

12.3. 'Negasystem state', '$ST_{\overline{\not S}}$', equals by definition 'set of conditions, $\underline{F}$, such that that negasystem, $\overline{\not S}$, such that that time, t, such that $\underline{F}$ at $\overline{\not S}$ at t '.

13. system property, $P_{\overline{S}}$

13.1. A system property is a system's conditions.

13.2. $P_{\overline{S}} =_{Df} \{\overline{S} \mid \underline{F}(\overline{S})\}$

13.3. 'System property', '$P_{\overline{S}}$', equals by definition 'set of systems, $\overline{S}$, such that condition, $\underline{F}$, at $\overline{S}$ '.

14. negasystem property, $P_{\overline{\not S}}$

14.1. A negasystem property is a negasystem's conditions.

14.2. $P_{\overline{\not S}} =_{Df} \{\overline{\not S} \mid \underline{F}(\overline{\not S})\}$

14.3. 'Negasystem property', '$P_{\overline{\not S}}$', equals by definition 'set of negasystems, $\overline{\not S}$, such that condition, $\underline{F}$, at $\overline{\not S}$ '.

15. value, V

16. system property state, $ST_{P_{\bar{S}}}$

16.1. A system property state is a system property's value at a given time.

16.2. $ST_{P_{\bar{S}}} =_{Df} \iota V \iota P_{\bar{S}} \iota t(V(P_{\bar{S}}(t)))$

16.3. 'System property state', '$ST_{P_{\bar{S}}}$', equals by definition 'that value, V, such that that system property, $P_{\bar{S}}$, such that that time, t, such that V at $P_{\bar{S}}$ at t '.

17. negasystem property state, $ST_{P_{\bar{\not{S}}}}$

17.1. A negasystem property state is a negasystem property's value at a given time.

17.2. $ST_{P_{\bar{\not{S}}}} =_{Df} \iota V \iota P_{\bar{\not{S}}} \iota t(V(P_{\bar{\not{S}}}(t)))$

17.3. 'Negasystem property state', '$ST_{P_{\bar{\not{S}}}}$', equals by definition 'that value, V, such that that negasystem property, $P_{\bar{\not{S}}}$, such that that time, t, such that V at $P_{\bar{\not{S}}}$ at t '.

18. system environmentness, $E_{\bar{S}}$

18.1. System environmentness is a negasystem of at least two components with at least one affect relation which has selective information.

18.2. $E_{\overline{S}} =_{Df} \{\overline{S} \mid n(\overline{\not{S}}) \geqq 2 \wedge \exists \mathcal{R}_A(\mathcal{R}_A \neq \emptyset \wedge \forall R_A(R_A \in \mathcal{R}_A \Rightarrow.$

$R_A \subset \overline{\not{S}} \times \overline{\not{S}} \wedge \exists \mathcal{I}_S(\forall I_S(I_S \in \mathcal{I}_S \Rightarrow. I_S \sim R_A \overline{\nabla} \exists \mathcal{R}(\mathcal{R} \subset R_A^{2^n} \wedge$

$I_S \sim \mathcal{R}) \overline{\nabla} \exists S'(S' \subset S \wedge I_S \sim S'))))))\}$

18.3. 'System environmentness', '$E_{\overline{S}}$', equals by definition 'set of systems, $\overline{S}$, such that the cardinality of the negasystem, $n(\overline{\not{S}})$, is greater than or equal to 2, and there is a family of affect relations, $\mathcal{R}_A$, such that $\mathcal{R}_A$ is not equal to the null set, $\emptyset$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}_A$, only if R_A is contained in the Cartesian product of $\overline{\not{S}}$ and $\overline{\not{S}}$, and there is a family of selective informations, $\mathcal{I}_S$, such that for all selective information, I_S, I_S is an element of $\mathcal{I}_S$ only if either I_S is equivalent to R_A or there is a family of relations, $\mathcal{R}$, such that $\mathcal{R}$ is contained in the power set of R_A and I_S is equivalent to $\mathcal{R}$ or there is a group, S', such that S' is contained in S and I_S is equivalent to S' and I_S is not equivalent to any combination thereof '.

19. negasystem environmentness, $E_{\overline{\not{S}}}$

19.1. Negasystem environmentness is a system with selective information

19.2. $E_{\overline{\not{S}}} =_{Df} \{\overline{\not{S}} \mid (\overline{S} \mid I_S(\overline{S}))(\overline{\not{S}})\}$

19.3. 'Negasystem environmentness', '$E_{\overline{\not{S}}}$', equals by definition 'set of negasystems, $\overline{\not{S}}$, such that the condition, system, $\overline{S}$, such that condition, selective information, I_S, at $\overline{S}$ at $\overline{\not{S}}$ '.

20. system environmental changeness, $EC_{\overline{S}}$

20.1. System environmental changeness is a difference in system environmentness.

20.2. $EC_{\overline{S}} =_{Df} \{\overline{S} \mid |ST_{E_{\overline{S}}}(t + \Delta t) - ST_{E_{\overline{S}}}(t)| \geqq \delta\}$

20.3. 'System environmental changeness', '$EC_{\overline{S}}$', equals by definition 'set of systems, $\overline{S}$, such that the absolute value of the system environmentness state, $ST_{E_{\overline{S}}}$, at time, t, plus an increment of t minus $ST_{E_{\overline{S}}}$ at t is greater than or equal to the real number, δ '.

21. negasystem environmental changeness, $EC_{\overline{\not S}}$

21.1. Negasystem environmental changeness is a difference in negasystem environmentness.

21.2. $EC_{\overline{\not S}} =_{Df} \{\overline{\not S} \mid |ST_{E_{\overline{\not S}}}(t + \Delta t) - ST_{E_{\overline{\not S}}}(t)| \geqq \delta\}$

21.3. 'Negasystem environmental changeness', '$EC_{\overline{\not S}}$', equals by definition 'set of negasystems, $\overline{\not S}$, such that the absolute value of the negasystem environmentness state, $ST_{E_{\overline{\not S}}}$, at time, t, plus an increment of t minus $ST_{E_{\overline{\not S}}}$ at t is greater than or equal to the real number, δ '.

22. toputness, TP

22.1. Toputness is system environmentness.

22.2. $TP =_{Df} E_{\overline{S}}$

22.3. 'Toputness', 'TP', equals by definition 'system environmentness, $E_{\overline{S}}$ '.

23. Inputness, IP

23.1. Inputness is a system with selective information.

23.2. $IP =_{Df} \{\bar{S} \mid I_S(\bar{S})\}$

23.3. 'Inputness', 'IP', equals by definition 'set of systems, $\bar{S}$, such that selective information, I_S, at $\bar{S}$ '.

24. fromputness, FP

24.1. Fromputness is negasystem environmentness.

24.2. $FP =_{Df} E_{\not{S}}$

24.3. 'Fromputness', 'FP', equals by definition 'negasystem environmentness, $E_{\not{S}}$ '.

25. outputness, OP

25.1. Outputness is a negasystem with selective information.

25.2. $OP =_{Df} \{\bar{\not{S}} \mid I_S(\bar{\not{S}})\}$

25.3. 'Outputness', 'OP', equals by definition 'set of negasystems, $\bar{\not{S}}$, such that selective information, I_S, at $\bar{\not{S}}$ '.

26. storeputness, SP

26.1. Storeputness is a system with inputness that is not fromputness.

26.2. $SP =_{Df} I_S^C(IP|FP)$ (3)

26.3. 'Storeputness', 'SP', equals by definition 'conditional selective information, I_S^C, at inputness, IP, given fromputness, FP '.

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[3] With respect to this system property and those to follow, only the conditions which define the properties will be stated.

27. feedinness, FI

27.1. Feedinness is transmission of selective information from a negasystem to a system.

27.2. $FI =_{Df} \vec{I}(TP,IP)$

27.3. 'Feedinness', 'FI', equals by definition 'transmission of selective information, $\vec{I}$, between toputness, TP, and inputness, IP '.

28. feedoutness, FO

28.1. Feedoutness is transmission of selective information from a system to a negasystem.

28.2. $FO =_{Df} \vec{I}(FP,OP)$

28.3. 'Feedoutness', 'FO', equals by definition 'transmission of selective information, $\vec{I}$, between fromputness, FP, and outputness, OP '.

29. feedthroughness, FT

29.1. Feedthroughness is transmission of selective information from a negasystem through a system to a negasystem.

29.2. $FT =_{Df} \vec{I}(TP,IP,FP,OP)$

29.3. 'Feedthroughness', 'FT', equals by definition 'transmission of selective information, $\vec{I}$, between toputness, TP, inputness, IP, fromputness, FP, and outputness, OP '.

30. feedbackness, FB

30.1. Feedbackness is transmission of selective information from a system through a negasystem to a system.

30.2. $FB =_{Df} \vec{I}(FP,OP,TP,IP)$

30.3. 'Feedbackness', 'FB', equals by definition 'transmission of selective information, $\vec{I}$, between fromputness, FP, outputness, OP, toputness, TP, and inputness, IP '.

31. filtrationness, FL

31.1. Filtrationness is a restriction of environmentness.

31.2. $FL =_{Df} |\max ST_{TP} - ST_{TP}| \geqq \delta$ [4]

31.3. 'Filtrationness', 'FL', equals by definition 'the absolute value of maximum toputness state, max ST_{TP}, minus toputness state, ST_{TP}, is greater than or equal to the real number, δ '.

32. spillageness, SL

32.1. Spillageness is a restriction of feedinness.

32.2. $SL =_{Df} |\max ST_{FI} - ST_{FI}| \geqq \delta$

32.3. 'Spillageness', 'SL', equals by definition 'the absolute value of maximum feedinness state, max ST_{FI}, minus feedinness state, ST_{FI}, is greater than or equal to the real number, δ '.

33. regulationness, RG

33.1. Regulationness is adjustment of fromputness.

33.2. $RG =_{Df} |ST_{FP}(t + \Delta t) - ST_{FP}(t)| \geqq \delta$

33.3. 'Regulationness', 'RG', equals by definition 'the absolute value of fromputness state, ST_{FP}, at time, t, plus an increment of t minus ST_{FP} at t is greater than or equal to the real number, δ '.

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[4] With respect to this system property state and those to follow, the indexing with '$\overline{S}$' will be omitted.

34. compatibleness, CP

34.1. Compatibleness is commonality between feedinness and feedoutness.

34.2. $CP =_{Df} B(FI, FO)$

34.3. 'Compatibleness', 'CP', equals by definition 'common information, B, at the pair of feedinness, FI, and feedoutness, FO '.

35. openness, O

35.1. Openness is feedinness and/or feedoutness.

35.2. $O =_{Df} ST_{FI} + ST_{FO} - ST_{CP} = \delta$

35.3. 'Openness', 'O', equals by definition 'feedinness state, ST_{FI} plus feedoutness state, ST_{FO}, minus compatibleness state, ST_{CP}, is equal to a real number, δ '.

36. adaptiveness, AD

36.1. Adaptiveness is a difference in compatibleness under system environmental changeness.

36.2. $AD =_{Df} |ST_{CP}(t + \Delta t) - ST_{CP}(t)| \geqq \delta \wedge EC_{\bar{S}}$

36.3. 'Adaptiveness', 'AD', equals by definition 'the absolute value of compatibleness state, ST_{CP}, at time, t, plus an increment of t minus ST_{CP} at t is greater than or equal to the real number, δ, and system environmental changeness, $EC_{\bar{S}}$ '.

37. efficientness, EF

37.1. Efficientness is commonality between feedthroughness and toputness.

37.2. $EF =_{Df} B(FT, TP)$

37.3. 'Efficientness', EF', equals by definition 'common information, B, at the pair of feedthroughness, FT, and toputness, TP '.

38. complete connectionness, CC

38.1. Complete connectionness is every two components directly channeled to each other with respect to affect relations.

38.2. $CC =_{Df} \exists \mathcal{R}'_A(\mathcal{R}'_A \subset \mathcal{R}_A \wedge \forall R_A(R_A \in \mathcal{R}'_A \Rightarrow R_A = R^D_{DA} \mid (s_j,s_i) \in R))$ (5)

38.3. 'Complete connectionness', 'CC', is defined as 'there is a family of affect relations, $\mathcal{R}'_A$, such that $\mathcal{R}'_A$ is contained in the family of affect relations, $\mathcal{R}_A$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}'_A$ only if R_A is equal to a direct directed affect relation, R^D_{DA}, such that the pair of component, s_j, and component, s_i, (s_j,s_i), is an element of relation, R '.

39. strongness, SR

39.1. Strongness is not complete connectionness and every two components are channeled to each other with respect to affect relations.

39.2. $SR =_{Df} \exists \mathcal{R}'_A(\mathcal{R}'_A \subset \mathcal{R}_A \wedge \forall R_A(R_A \in \mathcal{R}'_A \Rightarrow. R_A \neq R^D_{DA} \wedge R_A =$

$R_{DA} \mid (s_j,s_i) \in R))$ (2)

39.3. 'Strongness', 'SR', equals by definition 'there is a family of affect relations, $\mathcal{R}'_A$, such that $\mathcal{R}'_A$, is contained in the family of affect relations, $\mathcal{R}_A$, and for all affect relations, RA, R_A is an element of $\mathcal{R}'_A$ only if R_A is not equal to a direct directed affect relation, R^D_{DA}, and R_A is equal to a directed affect relation, R_{DA}, such that the pair of component, s_j, and component, s_i, (s_j,s_i), is an element of relation, R '.

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[5] The condition is a conjunct to the condition in 7-1-1.2.

40. unilateralness, U

40.1. Unilateralness is not either complete connectionness or strongness and every two components have a channel between them with respect to affect relations.

40.2. $U =_{Df} \exists \mathcal{R}'_A(\mathcal{R}'_A \subset \mathcal{R}_A \wedge \forall R_A(R_A \in \mathcal{R}'_A \Rightarrow R_A = R_{DA} \mid (s_j, s_i) \not\in R))$ (2)

40.3. 'Unilateralness', 'U', equals by definition 'there is a family of affect relations, $\mathcal{R}'_A$, such that $\mathcal{R}'_A$ is contained in the family of affect relations, $\mathcal{R}_A$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}'_A$ only if R_A is equal to a directed affect relation, R_{DA}, such that the pair of component, s_j, and component, s_i, (s_j, s_i), is not an element of relation, R '.

41. weakness, WE

41.1. Weakness is not either complete connectionness or strongness or unilateralness and every two components are connected with respect to affect relations.

41.2. $WE =_{Df} \exists \mathcal{R}'_A(\mathcal{R}'_A \subset \mathcal{R}_A \wedge \forall R_A(R_A \in \mathcal{R}'_A \Rightarrow R_A \neq R_{DA}))$

41.3. 'Weakness', 'WE', equals by definition 'there is a family of affect relations, $\mathcal{R}'_A$, such that $\mathcal{R}'_A$ is contained in the family of affect relations, $\mathcal{R}_A$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}'_A$ only if R_A is not equal to a directed affect relation, R_{DA} '.

42. disconnectionness, DC

42.1. Disconnectionness is not either complete connectionness or strongness or unilateralness or weakness and some components are not connected with respect to affect relations.

42.2. $DC =_{Df} \exists \mathcal{R}'_A(\mathcal{R}'_A \subset \mathcal{R}_A \wedge \forall R_A(R_A \in \mathcal{R}'_A \Rightarrow \exists s_i \exists s_j((s_i, s_j) \notin R_A)))$

42.3. 'Disconnectionness', 'DC', equals by definition 'there is a family of affect relations, $\mathcal{R}'_A$, such that $\mathcal{R}'_A$ is contained in the family of affect relations, $\mathcal{R}_A$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}'_A$ only if there is a component, s_i, such that there is a component, s_j, such that the pair of s_i and s_j, (s_i, s_j), is not an element of R_A '.

43. vulnerableness, VN

43.1. Vulenrableness is some connections which when removed produce disconnectionness with respect to affect relations.

43.2. $VN =_{Df} \exists \mathcal{R}'_A(\mathcal{R}'_A \subset \mathcal{R}_A \wedge \forall R_A(R_A \in \mathcal{R}'_A \Rightarrow . \, R_A = R \mid R \subset S \times S \wedge$

$\exists R'(R' \subset R \wedge DC(S|R - R')))$

43.3. 'Vulnerableness', 'VN', equals by definition 'there is a family of affect relations, $\mathcal{R}'_A$, such that $\mathcal{R}'_A$ is contained in the family of affect relations, $\mathcal{R}_A$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}'_A$ only if R_A is equal to a relation, R, such that R is contained in the Cartesian product of the group, S, and S, and there is a relation, R', such that R' is contained in R and the condition of disconnectionness, DC, at S such that R minus R' '.

44. passive dependentness, D_P

44.1. Passive dependentness is components which have channels to them.

44.2. $D_P =_{Df} \exists A(A \subset S \wedge \forall s(s \in A \Rightarrow \overline{T}_O(s) \neq \emptyset))$

44.3. 'Passive dependentness', 'D_P', equals by definition 'there is a set, A, such that A is contained in the group, S, and for all components, s, s is an element of A, only if the inverse transitive closure, $\overline{T}_O$, at s is not equal to the null set, $\emptyset$ '.

45. active dependentness, D_A

45.1. Active dependentness is components which have channels from them.

45.2. $D_A =_{Df} \exists A(A \subset S \wedge \forall s(s \in A \Rightarrow \Gamma_o(s) \neq \emptyset))$

45.3. 'Active dependentness', 'D_A', equals by definition 'there is a set, A, such that A is contained in the group, S, and for all components, s, s is an element of A only if the transitive closure, Γ_o, at s is not equal to the null set, $\emptyset$ '.

46. independentness, I

46.1. Independentness is components which do not have channels to them.

46.2. $I =_{Df} \exists A(A \subset S \wedge A \neq S \wedge \forall s(s \in A \Rightarrow \overline{\Gamma}_o(s) = \emptyset))$

46.3. 'Independentness', 'I', equals by definition 'there is a set, A, such that A is contained in the group, S, and A is not equal to S, and for all components, s, s is an element of A only if the inverse transitive closure, $\overline{\Gamma}_o$, at s is equal to the null set, $\emptyset$ '.

47. segregationness, SG

47.1. Segregationness is independentness under system environmental changeness.

47.2. $SG =_{Df} |ST_I(t + \Delta t) - ST_I(t)| \leqq \delta \wedge EC_{\overline{S}}$

47.3. 'Segregationness', 'SG', equals by definition 'the absolute value of independentness state, ST_I, at time, t, plus an increment of t, minus ST_I at t is less than or equal to the real number, δ, and system environmental changeness, $EC_{\overline{S}}$ '.

48. interdependentness, ID

48.1. Interdependentness is components which have channels to and from them.

48.2. $ID =_{Df} \exists A(A \subset S \wedge \forall s(s \in A \Rightarrow. \Gamma_o(s) \neq \emptyset \wedge \overline{\Gamma}_o(s) \neq \emptyset))$

48.3. 'Interdependentness', 'ID', equals by definition 'there is a set, A, such that A is contained in the group, S, and for all components, s, s is an element of A, only if the transitive closure, Γ_o, at s is not equal to the null set, $\emptyset$, and the inverse transitive closure, $\overline{\Gamma}_o$, at s is not equal to $\emptyset$ '.

49. wholeness, W

49.1. Wholeness is components which have channels to all other components.

49.2. $W =_{Df} \exists A(A \subset S \wedge \forall s_i(s_i \in A \Rightarrow \forall s_j(s_j \neq s_i \Rightarrow s_j \in \Gamma_o(s_i))))$

49.3. 'Wholeness', 'W', equals by definition 'there is a set, A, such that A is contained in the group, S, and for all components, s_i, s_i is an element of A only if for all components, s_j, s_j is not equal to s_i only if s_j is an element of the transitive closure, Γ_o, at s_i '.

50. Integrationness, IG

50.1. Integrationness is wholeness under system environmental changeness.

50.2. $IG =_{Df} |ST_W(t + \Delta t) - ST_W(t)| \leqq \delta \wedge EC_{\overline{S}}$

50.3. 'Integrationness', 'IG', equals by definition, 'the absolute value of wholeness state, ST_W, at time, t, plus an increment of t minus ST_W at t is less than or equal to the real number, δ, and system environmental changeness, $EC_{\overline{S}}$ '.

51. hierarchically orderness, HO

51.1. Hierarchically orderness is levels of subordinateness with components in each level with respect to affect relations.

51.2. $$HO =_{Df} \exists \mathcal{R}_{DA}(\mathcal{R}_{DA} \subset \mathcal{R}_A \wedge \forall R_A(R_A \in \mathcal{R}_{DA} \Rightarrow R_A = (\bigcup_{i=1}^{n} R_i) \cup (\bigcup_{i=1}^{n+1} R'_i) \wedge \bigwedge_{i=1}^{n}(R_i \cap R_{i+1} = R'_i \cap R'_{i+1} = R_i \cap R'_i = \emptyset) \wedge \bigwedge_{i=1}^{n+1}(R'_i = \bigcup_{j=1}^{m} R'_j \wedge SR(R'_j)) \wedge \bigwedge_{i=1}^{n}(D(R_i) \subset R(R'_i) \wedge R(R_i) \subset D(R'_{i+1}) \wedge R_i \neq \emptyset)))$$

51.3. 'Hierarchically orderness', 'HO', equals by definition 'there is a family of directed affect relations, $\mathcal{R}_{DA}$, such that $\mathcal{R}_{DA}$ is contained in the family of affect relations, $\mathcal{R}_A$, and for all affect relations, R_A, R_A is an element of $\mathcal{R}_{DA}$, only if R_A is equal to the union of the union of relations, R_i, where R is indexed from 1 to n and the union of relations, R'_i, where R′ is indexed from 1 to n+1, and the conjunction of: the intersection of R_i and R_{i+1} is equal to the intersection of R'_i and R'_{i+1} is equal to the intersection of R_i and R'_i is equal to the null set, $\emptyset$; where variables of the conjuncts are indexed from 1 to n, and the conjunction of: R'_i is equal to the union of relations, R'_j, where R′ is indexed from 1 to m and strongness, SR, at R'_j; where variables of the conjuncts are indexed from 1 to n+1, and the conjunction of: the domain, D, at R_i is contained in range, R, at R'_i and R at R_i is contained in D at R'_{i+1} and R_i is not equal to $\emptyset$; where variables of the conjuncts are indexed from 1 to n '.

52. flexibleness, F

52.1. Flexibleness is different subgroups of components through which there is a channel between two components with respect to affect relations.

52.2. $\mathcal{F} =_{Df} \exists \mathcal{R}_{DA}(\mathcal{R}_{DA} \subset \mathcal{R}_{A} \wedge \forall R_{A}(R_{A} \in \mathcal{R}_{DA} \Rightarrow \exists \mathcal{S}(\forall s_{i}(s_{i} \in S \Rightarrow \forall s_{j}(s_{j} \in S \wedge (s_{i},s_{j}) \in R_{A} \mathrel{.\!\Rightarrow} \exists S'(S' \in \mathcal{S} \wedge \exists S''(S'' \in \mathcal{S} \wedge S' \cap S'' = \{s_{i},s_{j}\} \wedge \exists m(m \geqq 1 \wedge \exists n(n > 1 \wedge s_{j} \in \Gamma^{m}_{S'}(s_{i}) \wedge s_{j} \in \Gamma^{n}_{S''}(s_{i}))))))))))$

52.3. 'Flexibleness', '$\mathcal{F}$', equals by definition 'there is a family of directed affect relations, $\mathcal{R}_{DA}$, such that $\mathcal{R}_{DA}$ is contained in the family of affect relations, $\mathcal{R}_{A}$, and for all affect relations, R_{A}, R_{A} is an element of $\mathcal{R}_{DA}$ only if there is a family of groups, $\mathcal{S}$, such that for all components, s_{i}, s_{i} is an element of group, S, only if for all components, s_{j}, s_{j} is an element of S and the pair of s_{i} and s_{j}, (s_{i},s_{j}), is an element of R_{A}, only if there is a group, S', such that S' is an element of $\mathcal{S}$, and there is a group, S'', such that S'' is an element of $\mathcal{S}$, and the intersection of S' and S'' is equal to the set of s_{i} and s_{j}, $\{s_{i},s_{j}\}$, and there is an integer, m, such that m is greater than or equal to 1, and there is an integer, n, such that n is greater than 1 and s_{j} is an element of gamma m, Γ^{m}, at s_{i} with respect to S', and s_{j} is an element of gamma n, Γ^{n}, at s_{i} with respect to S'' '.

53. homomorphismness, HM

53.1. Homomorphismness is components having the same connections as other components.

53.2. $HM =_{Df} \exists S'(S' \subset S \wedge \exists S''(S'' \subset S \wedge \exists \beta(\beta \mid S' \rightarrow S'')))$

53.3. 'Homomorphismness', 'HM', equals by definition 'there is a group, S', such that S' is contained in the group, S, and there is a group, S'', such that S'' is contained in S, and there is a homomorphic mapping, β, such that β is defined from S' onto S'' '.

54. Isomorphismness, IM

54.1. Isomorphismness is components having the same connections as other corresponding components.

54.2. $\mathrm{IM} =_{\mathrm{Df}} \exists S'(S' \subset S \wedge \exists S''(S'' \subset S \wedge \exists \alpha(\alpha \mid S' \rightarrow S'')))$

54.3. 'Isomorphismness', 'IM', equals by definition 'there is a group, S', such that S' is contained in the group, S, and there is a group, S'', such that S'' is contained in S, and there is an isomorphic mapping, α, such that α is defined from S' onto S'' '.

55. automorphismness, AM

55.1. Automorphismness is components whose connections can be transformed so that the same connections hold.

55.2. $\mathrm{AM} =_{\mathrm{Df}} \exists S'(S' \subset S \wedge \exists \alpha(\alpha \mid S' \rightarrow S'))$

55.3. 'Automorphismness', 'AM', equals by definition 'there is a group, S', such that S' is contained in the group, S, and there is an isomorphic mapping, α, such that α is defined from S' into S' '.

56. compactness, CO

56.1. Compactness is average number of direct channels in a channel between components.

56.2. $$\mathrm{CO} =_{\mathrm{Df}} \exists p(\exists s_i(s_i \in S \wedge \exists s_j(s_j \in S \wedge \forall s_k(s_k \in S \Rightarrow \forall s_m(s_m \in S \Rightarrow.$$

$$d(s_i,s_j) \geqq d(s_k,s_m) \wedge \exists n(n = ST_{SZ} \wedge \frac{\sum\limits_{\substack{k=1\\m=1\\k\neq m}}^{n} d(s_i,s_j) - d(s_k,s_m)}{n^2 - n} =$$

$$p))))))$$

56.3. 'Compactness', 'CO', equals by definition 'there is a probability, p, such that there is a component, s_i, such that s_i is an element of group, S, and there is a component, s_j, such that s_j is an element of S and for all components, s_k, s_k is an element of S, only if for all components, s_m, s_m is an element of S, only if the distance from s_i to s_j, $d(s_i,s_j)$, is greater than or equal to the distance from s_k to s_m, $d(s_k,s_m)$, and there is a number, n, such that n is equal to sizeness state, ST_{SZ}, and the summation from k is equal to 1 to n and m is equal to 1 to n and k is not equal to m, of $d(s_i,s_j)$ minus $d(s_k,s_m)$ divided by n squared minus n is equal to p '.

57. centralness, CE

57.1. Centralness is concentration of channels.

57.2. $CE =_{Df} \exists A(A \subset S \wedge \forall B(B \subset S \Rightarrow \exists \mathcal{R}_{DA}(\mathcal{R}_{DA} \subset \mathcal{R}_A \wedge \forall R_{DA}(R_{DA} \in \mathcal{R}_{DA} \Rightarrow$
$\Delta_{oR_{DA}}(B) \subset \Delta_{oR_{DA}}(A)))))$

57.3. 'Centralness', 'CE', equals by definition 'there is a set, A, such that A is contained in group, S, and for all sets, B, B is contained in S only if there is a family of directed affect relations, $\mathcal{R}_{DA}$, such that $\mathcal{R}_{DA}$ is contained in the family of affect relations, $\mathcal{R}_A$, and for all directed affect relations, R_{DA}, R_{DA} is an element of $\mathcal{R}_{DA}$ only if the affect function, Δ_o, at B with respect to R_{DA} is contained in Δ_o at A with respect to R_{DA} '.

58. sizeness, SZ

58.1. Sizeness is the number of components.

58.2. $SZ =_{Df} n(\{s_1,\ldots,s_n\})$

58.3. 'Sizeness', 'SZ', equals by definition 'the cardinality of the set of components, s_1 through s_n '.

59. complexness, CX

59.1. Complexness is the number of connections.

59.2. $CX =_{Df} n(\bigcup_{R_A \in \mathcal{R}_A} R_A)$

59.3. 'Complexness', 'CX', equals by definition 'the cardinality of the union of the affect relations, R_A, as R_A varies over the family of affect relations, $\mathcal{R}_A$ '.

60. selective informationness, SI

60.1. Selective informationness is amount of selective information.

60.2. $SI =_{Df} \sum_{c \in T_S} p(c) \log \frac{1}{p(c)}$

60.3. 'Selective informationness', 'SI', equals by definition 'summation of probability, p, at category, c, times the logarithm, log, of 1 divided by p at c where c varies over I_S '.

61. size growthness, ZG

61.1. Size growthness is increase in sizeness.

61.2. $ZG =_{Df} ST_{SZ}(t + \Delta t) \geqq ST_{SZ}(t)$

61.3. 'Size growthness', 'ZG', equals by definition 'sizeness state, ST_{SZ}, at time, t, plus an increment of t is greater than or equal to ST_{SZ} at t '.

62. complexity growthness, XG

62.1. Complexity growthness is increase in complexness.

62.2. $XG =_{Df} ST_{CX}(t + \Delta t) \geqq ST_{CX}(t)$

62.3. 'Complexity growthness', 'XG', equals by definition 'complexness state, ST_{CX}, at time, t, plus an increment of t is greater than or equal to ST_{CX} at t '.

63. selective information growthness, TG

63.1. Selective information growthness is increase in selective informationness.

63.2. $TG =_{Df} ST_{SI}(t + \Delta t) \geqq ST_{SI}(t)$

63.3. 'Selective information growthness', 'TG', equals by definition 'selective informationness state, ST_{SI}, at time, t, plus an increment of t is greater than or equal to ST_{SI} at t '.

64. size degenerationness, ZD

64.1. Size degenerationness is decrease in sizeness.

64.2. $ZD =_{Df} ST_{SZ}(t + \Delta t) \leqq ST_{SZ}(t)$

64.3. 'Size degenerationness', 'ZD', equals by definition 'sizeness state, ST_{SZ}, at time, t, plus an increment of t is less than or equal to ST_{SZ} at t '.

65. complexity degenerationness, XD

65.1. Complexity degenerationness is decrease in complexness.

65.2. $XD =_{Df} ST_{CX}(t + \Delta t) \leqq ST_{CX}(t)$

65.3. 'Complexity degenerationness', 'XD', equals by definition 'complexness state, ST_{CX}, at time, t, plus an increment of t is less than or equal to ST_{CX} at t '.

66. selective information degenerationness, TD

66.1. Selective information degenerationness is decrease in selective informationness.

66.2. $TD =_{Df} ST_{SI}(t + \Delta t) \leqq ST_{SI}(t)$

66.3. 'Selective information degenerationness', 'TD', equals by definition 'selective informationness state, ST_{SI}, at time, t, plus an increment of t is less than or equal to ST_{SI} at t '.

67. stableness, SB

67.1. Stableness is no change with respect to conditions.

67.2. $SB =_{Df} ST_{\overline{S}}(t_1) \cap ST_{\overline{S}}(t_2) \neq \emptyset$

67.3. 'Stableness', 'SB', equals by definition 'the intersection of system state, $ST_{\overline{S}}$, at time, t_1, and $ST_{\overline{S}}$ at time, t_2, is not equal to the null set, $\emptyset$ '.

68. state steadiness, SS

68.1. State steadiness is stableness under system environmental change.

68.2. $SS =_{Df} |ST_{SB}(t + \Delta t) - ST_{SB}(t)| \leqq \delta \wedge EC_{\overline{S}}$

68.3. 'State steadiness', 'SS', equals by definition 'the absolute value of stableness state, ST_{SB}, at time, t, plus an increment of t minus ST_{SB} at t is less than or equal to real number, δ, and system environmental changeness, $EC_{\overline{S}}$ '.

69. state determinationness, SD

69.1. State determinationness is derivability of conditions from one and only one state.

69.2. $SD =_{Df} \exists \mathcal{ST}_{\overline{S}}(\forall ST'_{\overline{S}}(ST'_{\overline{S}} \in \mathcal{ST}_{\overline{S}} \wedge ST'_{\overline{S}}(t + \Delta t) \ .\!\Rightarrow \exists A(A \subset ST'_{\overline{S}} \wedge$

$\iota ST''_{\overline{S}}(ST''_{\overline{S}} \in \mathcal{ST}_{\overline{S}} \wedge ST''_{\overline{S}}(t) \wedge ST''_{\overline{S}} \vdash A))))$

69.3. 'State determinationness', 'SD', equals by definition 'there is a family of system states, $\mathcal{ST}_{\overline{S}}$, such that for all system states, $ST'_{\overline{S}}$, $ST'_{\overline{S}}$ is an element of $\mathcal{ST}_{\overline{S}}$ and $ST'_{\overline{S}}$ at time, t, plus an increment of t only if there is a set, A, such that A is contained in $ST'_{\overline{S}}$ and that system state, $ST''_{\overline{S}}$, such that $ST''_{\overline{S}}$ is an element of $\mathcal{ST}_{\overline{S}}$ and $ST''_{\overline{S}}$ at t and $ST''_{\overline{S}}$ yields A '.

70. equifinalness, EL

70.1. Equifinalness is derivability of conditions from other states.

70.2. $EL =_{Df} \exists \mathcal{ST}'_{\overline{S}}(\forall ST'_{\overline{S}}(ST'_{\overline{S}} \in \mathcal{ST}'_{\overline{S}} \wedge ST'_{\overline{S}}(t + \Delta t) \ .\!\Rightarrow \exists A(A \subset ST'_{\overline{S}} \wedge \exists \mathcal{ST}''_{\overline{S}}(\mathcal{ST}''_{\overline{S}} = \{ST''_{\overline{S}_i} \mid 1 \leqq i \wedge i \leqq n \wedge n \geqq 2\} \wedge \forall ST''_{\overline{S}}(ST''_{\overline{S}} \in \mathcal{ST}''_{\overline{S}} \wedge ST''_{\overline{S}}(t) \ .\!\Rightarrow ST''_{\overline{S}} \vdash A)))))$

70.3. 'Equifinalness', 'EL', equals by definition 'there is a family of system states, $\mathcal{ST}'_{\overline{S}}$, such that for all system states, $ST'_{\overline{S}}$, $ST'_{\overline{S}}$ is an element of $\mathcal{ST}'_{\overline{S}}$ and $ST'_{\overline{S}}$ at time, t, plus an increment of t, only if there is a set, A, such that A is contained in $ST'_{\overline{S}}$ and there is a family of system states, $\mathcal{ST}''_{\overline{S}}$, such that $\mathcal{ST}''_{\overline{S}}$ is equal to a set of system states, $ST''_{\overline{S}_i}$, such that 1 is less than or equal to i and i is less than or equal to n and n is greater than or equal to 2, and for all system states, $ST''_{\overline{S}}$, $ST''_{\overline{S}}$ is an element of $\mathcal{ST}''_{\overline{S}}$ and $ST''_{\overline{S}}$ at t, only if $ST''_{\overline{S}}$ yields A '.

71. homeostasisness, HS

71.1. Homeostasisness is equifinalness under system environmental changeness.

71.2. $HS =_{Df} |ST_{EL}(t + \Delta t) - ST_{EL}(t)| \leqq \delta \wedge EC_{\overline{S}}$

71.3. 'Homeostasisness', 'HS', equals by definition 'the absolute value of equifinalness state, ST_{EL}, at time, t, plus an increment of t minus ST_{EL} at t is less than or equal to real number, δ, and system environmental changeness, $EC_{\overline{S}}$ '.

72. stressness, SE

72.1. Stressness is change beyond certain limits of negasystem state.

72.2. $SE =_{Df} |ST_{\overline{\cancel{S}}}(t + \Delta t) - ST_{\overline{\cancel{S}}}(t)| \geqq \delta$

72.3. 'Stressness', 'SE', equals by definition 'the absolute value of the negasystem state, $ST_{\overline{\cancel{S}}}$, at time, t, plus an increment of t minus $ST_{\overline{\cancel{S}}}$, at t is greater than or equal to the real number, δ '.

73. strainness, SA

73.1. Strainness is change beyond certain limits of system state.

73.2. $SA =_{Df} |ST_{\bar{S}}(t + \Delta t) - ST_{\bar{S}}(t)| \geqq \delta$

73.3. 'Strainness', 'SA', equals by definition 'the absolute value of the system state, $ST_{\bar{S}}$, at time, t, plus an increment of t minus $ST_{\bar{S}}$ at t is greater than or equal to the real number, δ '.

The characterizations in the model are of two kinds: primitive and defined terms which do not directly characterize general systems but which are required to do so, and defined terms (most of which are properties) which directly characterize general systems. Table 1 is a list of the former, while Table 2 is a list of the latter. These tables are presented on the following pages.

INDIRECT SYSTEM CHARACTERIZATIONS

PRIMITIVE

1. universe of discourse, $\mathcal{U}$

2. component, s

4. characterization, CH

10. condition, $\underline{F}$

15. value, V

DEFINED

3. group, S

5. information, I

5-1. selective information, I_S

5-1-1. nonconditional selective information, I_S^N

5-1-2. conditional selective information, I_S^C

6. transmission of selective information, $\vec{T}(I_{S_1}, I_{S_2}, \ldots, I_{S_i}, \ldots, I_{S_n})$

7. affect relation, R_A

7-1. directed affect relation, R_{DA}

7-1-1. direct directed affect relation, R_{DA}^D

7-1-2. indirect directed affect relation, R_{DA}^I

9. negasystem, $\overline{\mathcal{S}}$

12. negasystem state, $ST_{\overline{\mathcal{S}}}$

14. negasystem property, $P_{\overline{\mathcal{S}}}$

17. negasystem property state, $ST_{P_{\overline{\mathcal{S}}}}$

19. negasystem environmentness, $E_{\overline{\mathcal{S}}}$

21. negasystem environmental changeness, $EC_{\overline{\mathcal{S}}}$

24. fromputness, FP

25. outputness, OP

Table 1

(19, 21, 24, and 25 are negasystem properties.)

DIRECT SYSTEM CHARACTERIZATIONS

NON-PROPERTIES

8. system, $\overline{S}$
11. system state, $ST_{\overline{S}}$
13. system property, $P_{\overline{S}}$
16. system property state, $ST_{P_{\overline{S}}}$

PROPERTIES

18. system environmentness, $E_{\overline{S}}$
20. system environmental changeness, $EC_{\overline{S}}$
22. toputness, TP
23. inputness, IP
26. storeputness, SP
27. feedinness, FI
28. feedoutness, FO
29. feedthroughness, FT
30. feedbackness, FB
31. filtrationness, FL
32. spillageness, SL
33. regulationness, RG
34. compatibleness, CP
35. openness, O
36. adaptiveness, AD
37. efficientness, EF
38. complete connectionness, CC
39. strongness, SR
40. unilateralness, U
41. weakness, WE
42. disconnectionness, DC
43. vulnerableness, VN
44. passive dependentness, D_P
45. active dependentness, D_A
46. independentness, I
47. segregationness, SG
48. interdependentness, ID
49. wholeness, W
50. integrationness, IG
51. hierarchically orderness, HO
52. flexibleness, F
53. homomorphismness, HM
54. isomorphismness, IM
55. automorphismness, AM
56. compactness, CO
57. centralness, CE
58. sizeness, SZ
59. complexness, CX
60. selective informationness, SI
61. size growthness, ZG
62. complexity growthness, XG
63. selective information growthness, TG
64. size degenerationness, ZD
65. complexity degenerationness, TD
66. selective information degenerationness, ID
67. stableness, SB
68. state steadiness, SS
69. state determinationness, SD
70. equifinalness, EL
71. homeostasisness, HS
72. stressness, SE
73. strainness, SA

Table 2

CHAPTER V

RELATION OF SET THEORY, INFORMATION THEORY, AND GRAPH THEORY TO GENERAL SYSTEMS THEORY

Relation in Literature

Of Set Theory

Only two explicit relatings of set theory to general systems theory were discovered, i.e. Ashby's and Mesarovic's. Rosen and Rashevsky, however, have related set theory to a kind of system, i.e. a biological system.

S-1. Ashby, W. Ross, "The Set Theory of Mechanism and Homeostasis," General Systems, Vol. 9, 1964, pp. 83-97.

Basic set theoretic terms are developed and used to characterize systems. The elements of a system are states. A mapping defined on a system characterizes the relations between states, and thus the structure of a system. A definition of 'system with parts' is given in which the elements of the parts are states and the states of the system are listings of states of parts. Moreover, it is pointed out that a more general definition of 'system' could be given in which transitions between states are not determinate but have well-defined probabilities.

S-2. Martinez, H. M., "Toward an Optimal Design Principle in Relational Biology," Bulletin of Mathematical Biophysics, Vol. 26, 1964, pp. 351-365.

"A vulnerability criterion is posed for a biological system within the context of representing the system as a relational set . . . ." (p. 351)

S-3. Mesarovic, Mihajlo D., "Foundations for a General Systems Theory," Views on General Systems Theory, ed. by Mesarovic, New York: John Wiley and Sons, 1964, pp. 1-24.

"A general system" is defined as "a proper subset, X_S, of the Cartesian product of n sets of values, $X_1, \ldots, X_n$, where each set

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[1] A brief annotation of each item of the relevant literature is included. Each item is numbered and lettered in order to permit cross-referencing, since some items relate more than one of the theories to general systems theory. The letter before the number sequencing the items indicates either set or information or graph theoretic related literature, i.e. S or I or G respectively.

specifies a formal object'. "An attribute of a system is a propositional function defined on X and valid in X_S." (p. 17) A closed system is one in which there is an effective identification of each element. A system which is not closed is open. Through a set theoretic decomposition procedure on the system relation, $R[X_1, \ldots, X_n]$, it is shown that "a higher order system cannot be decomposed into subsystems with less than triadic relations" (p. 15). "The three terms of the triadic relation are, then, input, output, and state." (p. 17) Set theory is used to characterize a controllable system as one in which the performance is determined given a set of inputs.

S-4. Rashevsky, N., "A Comparison of Set-Theoretical and Graph-Theoretical Approaches in Topological Biology," Bulletin of Mathematical Biophysics, Vol. 20, 1958, pp. 267-280.

Through the addition of a postulate, the set theoretic approach to biology is extended to handle certain combinatorial relations suggested by graph theory.

S-5. Rashevsky, N., "A Note on the Nature and Origin of Life," Bulletin of Mathematical Biophysics, Vol. 21, 1959, pp. 185-193.

"A . . . study of the relational properties of . . . systems seems to offer the possibility of deriving the principle of biological mapping from the requirement of self reproduction and adaptability." (p. 185)

S-6. Rashevsky, N., "Contributions to Relational Biology," Bulletin of Mathematical Biophysics, Vol. 22, 1960, pp. 73-84.

The merit of the graph theoretic approach to biology and Rosen's characterization of a biological system in which the vertices of the graph are biological functions (S-10) is discussed.

S-7. Rashevsky, N., "On Relations between Sets," Bulletin of Mathematical Biophysics, Vol. 23, 1961, pp. 233-235.

"With a view to future applications in relational biology, the notation of relations between sets is introduced and several theorems are demonstrated." (p. 233)

S-8. Rashevsky, N., "Abstract Mathematical Molecular Biology," Bulletin of Mathematical Biophysics, Vol. 23, 1961, pp. 237-260.

". . . the problem of the minimal size of a living unit is studied . . . both from a metric and from a relational point of view." (p. 237)

S-9. Rosen, Robert, "A Relational Theory of Biological Systems," Bulletin of Mathematical Biophysics, Vol. 20, 1958, pp. 245-260.

Using set and graph theoretic approaches, a relational theory for metabolizing systems which explains a number of diverse phenomena, e.g. encystment and the existence of the cell nucleus, is set forth.

S-10. Rosen, Robert, "The Representation of Biological Systems from the Standpoint of the Theory of Categories," Bulletin of Mathematical Biophysics, Vol. 20, 1958, pp. 317-341.

Using functions to represent components and Cartesian products to represent inputs and outputs, a unique representation for each biological system is exhibited. The representation of the finite automaton is constructed.

S-11. Rosen, Robert, "A Relational Theory of Biological Systems II," Bulletin of Mathematical Biophysics, Vol. 21, 1959, pp. 109-128.

The characterization in S-10 of a biological system is extended. A set of axioms characterizing $(\mathfrak{M}, \mathfrak{R})$-systems is posited, and a principle of optimal design is proposed.

S-12. Rosen, Robert, "A Relational Theory of the Structure Induced in Biological Systems by Alterations in Environment," Bulletin of Mathematical Biophysics, Vol. 23, 1961, pp. 165-171.

"It is shown that a wide variety of structural alterations in both the "metabolic" and "genetic" apparatus of $(\mathfrak{M}, \mathfrak{R})$-systems can result from specific changes in the environment of such systems." (p. 165)

S-13. Rosen, Robert, "A Note on Abstract Relational Biologies," Bulletin of Mathematical Biophysics, Vol. 24, 1962, pp. 31-38.

"It is shown that the class of abstract block diagrams of $(\mathfrak{M}, \mathfrak{R})$-systems which can be constructed out of the objects and mapping of a particular subcategory $\mathfrak{C}_0$ of the category $\mathfrak{C}$ of all acts depends heavily on the structure of $\mathfrak{C}_0$ . . ." (p. 31)

S-14. Rosen, Robert, "On the Reversibility of Environmentally Induced Alterations in Abstract Biological Systems," Bulletin of Mathematical Biophysics, Vol. 25, 1963, pp. 41-50.

". . . environmentally induced alterations in structure of $(\mathfrak{M}, \mathfrak{R})$-systems [S-12] . . . are examined from the standpoint of determining under what circumstances they can be reversed by further environmental interactions." (p. 41)

S-15. Rosen, Robert, "Some Results in Graph Theory and their Application to Abstract Relational Biology," Bulletin of Mathematical Biophysics, Vol. 25, 1963, pp. 231-241.

> ". . . the problem of characterizing those categories suitable for a rich theory of $(\mathfrak{M}, \mathfrak{R})$-systems reduces to a problem familiar from the general theory of graphs." (p. 231)

S-16. Rosen, Robert, "Abstract Biological Systems as Sequential Machines," Bulletin of Mathematical Biophysics, Vol. 26, 1964, pp. 103-111.

> "The purpose of this note is to point out certain similarities which exist between the theory of sequential machines, . . . and the theory of $(\mathfrak{M}, \mathfrak{R})$-systems . . ." (p. 103)

S-17. Rosen, Robert, "Abstract Biological Systems as Sequential Machines II: Strong Connectedness and Reversibility," Bulletin of Mathematical Biophysics, Vol. 26, 1964, pp. 239-246.

> "The reversibility of environmentally induced structural changes in these systems is closely related to the strong connectedness of the corresponding machines." (p. 239)

Of Information Theory

Even though there is an extensive literature in which information theory is applied, only a small number of items are included. The reason for exclusion of the other items is lack of indication of the relation of information theory to either a system in general or a particular kind of system provided it can be considered in the context of a system in general.

Most of the articles dealing with the application of information theory which are found in publications of the Institutes of Radio Engineers (IRE) and of Electrical and Electronics Engineers (IEEE) assume a communication context and further within this context do not make explicit how information theory is related to a system. Consequently, these articles are excluded. Those articles which are within a systems engineering context are excluded also. In systems engineering the concern is with specific systems not with a system in general or kinds of systems. The emphasis is not theoretical but practical, i.e. the emphasis is on the arrangement and selection of components in a given system for efficient realization of a given outcome or outcomes. Moreover, those articles which are solely within a cybernetics context are excluded. As a justification of this exclusion, note von Bertalanffy's conclusion that cybernetics is a specific case of general systems theory:

> . . . the ultimate reason of the pattern and order in living systems can be sought only in the laws of the process itself, not in preestablished enduring structures . . . . "Dynamics" is the broader theory since we can come, from general system principles, always to regulations by machines, introducing conditions of constraint, but not vice versa.[2]

Most of the literature in psychology is ruled out due to a treatment of information theory within a measurement context rather than a theoretic one.

I-1. Berlyne, D. E., "Uncertainty and Conflict: A Point of Contact Between Information Theory and Behavior Theory Concepts," Psychological Review, Vol. 64, 1957, pp. 329-339.

> "The "uncertainty" function satisfies [with modifications] some of the requirements . . . for a measure of "degree of conflict" . . . . A discussion of . . . variables that appear to depend on degree of conflict reveals . . . links with information theory." (p. 339)

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[2] "General Systems Theory: A New Approach to Unity in Science, 6. Towards a Physical Theory of Organic Teleology, Feedback and Dynamics," Human Biology, Vol 23, 1951, pp. 360-361.

I-2. Bertalanffy, Ludwig von, "General System Theory: A New Approach to Unity in Science, 6. Towards a Physical Theory of Organic Teleology, Feedback and Dynamics," *Human Biology*, Vol. 23, 1951, pp. 346-361.

Cybernetics and general systems theory are compared and it is concluded that ". . . the ultimate reason of the pattern and order in the living system can be sought only in the laws of the process itself, not in pre-established and enduring structures . . . . "Dynamics" is the broader theory since we can come, from general system principles, always to regulation by machines, introducing conditions of constraint, but not *vice versa*" (p. 361).

I-3. Bertalanffy, Ludwig von, "General System Theory," *General Systems* Vol. 1, 1956, pp. 1-10.

". . . negative entropy of information is a measure of order or of organization since the latter, compared to distribution at random is an improbable state. In this way information theory comes close to the theory of open systems, which may increase in order and organization, or show negative entropy." (p. 5)

I-4. Bertalanffy, Ludwig von, "General System Theory--A Critical Review," *General Systems*, Vol. 7, 1962, pp. 1-20.

It is pointed out that information theory introduces information as a quantity measurable by an expression isomorphic to negative entropy in physics. Relationships of entropy, negative entropy, and channel capacity of systems to other system properties are given.

I-5. Boulding, Kenneth, "General Systems Theory--The Skeleton of Science," *General Systems*, Vol. 1, 1956, pp. 11-17.

"At the biological level . . . the information concept may serve to develop general notions of structuredness and abstract measures of organization which give us . . . a third basic dimension beyond mass and energy. . . . Information processes are . . . unquestionably essential in the development of organization, both in the biological and the social world." (p. 14)

I-6. Bremermann, H. J., "Optimization through Evolution and Recombination," *Self-Organizing Systems*, ed. by Marshall C. Yovits, George T. Jacobi, and Gordon D. Goldstein, Washington, D. C.: Spartan Books, 1962, pp. 93-106.

A conjecture is made which is derived from an argument based on quantum theory considerations: "No data processing system whether artificial or living can process more than (2×10^{47}) bits per second per gram of its mass" (p. 93).

I-7. Brillouin, L., "Physical Entropy and Information. II," Journal of Applied Physics, Vol. 22, 1951, pp. 338-343.

"The laws of statistical thermodynamics are used for the definition of entropy, and it is shown that the definition of information can be reduced to a problem of Fermi-Dirac statistics or to a generalized Fermi statistics. With these definitions, the entropy of a certain message can be defined, and the information contained in the message can be directly connected with the decrease of entropy in the system. This definition leads directly to the formulas proposed by C. E. Shannon for the measure of information, and shows that Shannon's "entropy of information" corresponds to an equal amount of negative entropy in the physical system." (p. 338)

I-8. Foerster, Heinz von, "On Self-Organizing Systems and Their Environments," Self-Organizing Systems, ed. by Marshall C. Yovits and Scott Cameron, New York: The Macmillan Company, 1959, pp. 31-50.

A self-organizing system is taken to be a system in which

$$H \frac{\delta H_{max}}{\delta t} > H_{max} \frac{\delta H}{\delta t}$$

where 'H' stands for the entropy of the set of elements defined in the representation of a system, and 'H_{max}' stands for the maximum possible entropy of this set.

I-9. Iri, Masao, "Theory of General Information Networks: An Algebraic Topological Foundation to the Theory of Information-Handling Systems," Mathematical Theory of Automata, ed. by Jerome Fox, New York: Polytechnic Press, 1963, pp. 415-435.

""Systems theory" involves a large number and variety of "network" problems. In general, a network problem has two aspects, one concerning the geometrical, or topological, structure of the underlying graph, and the other concerning the algebraic structure of the quantities associated with, or superimposed upon, the elements of the graph. Thus, we can regard a network as a system processing "informations" (which are the superimposed quantities) on a graph in regard to the specific algebraic character of the problem." (p. 415)

I-10. Jacobson, H., "Information, Reproduction, and the Origin of Life," American Scientist, Vol. 43, 1955, pp. 119-127.

Information theory is used as a means for analyzing life reproduction processes and as a complexity measure; e.g. "to estimate the minimal complexity of the first living organism" (p. 119).

I-11. Jaynes, E. T., "Information Theory and Statistical Mechanics, I," The Physical Review, Vol. 106, 1957, pp. 620-630.

Information theory provides a basis for statistical mechanics so that it "need not be regarded as a physical theory dependent for its validity on the truth of additional assumptions not contained in the laws of mechanics" (p. 620).

I-12. Jaynes, E. T., "Information Theory and Statistical Mechanics, II," The Physical Review, Vol. 108, 1957, pp. 171-190.

". . . a general theory of irreversible processes cannot be based on differential rate equations corresponding to time-proportional transition probabilities." (p. 171)

I-13. Karreman, George, "Topological Information Content and Chemical Reactions," Bulletin of Mathematical Biophysics, Vol. 17, 1955, pp. 279-287.

The topological information content of a graph (I-28) is applied to several topological types of chemical reactions, and so the influence of the structure of the chemical compounds in a reaction is illustrated.

I-14. Kochen, M., "An Information-Theoretic Model of Organizations," IEEE Transactions on Information Theory, Vol. IT-4, 1954, pp. 67-76.

A theory based on information theory is presented in order to give precision to the concept of the efficiency of a system.

I-15. Leibovic, K. N., "Information Processing Organs, Mathematical Mappings and Self-Organizing Systems," Bulletin of Mathematical Biophysics, Vol. 25, 1963, pp. 189-201.

"In this paper, some mathematical aspects of transformations and classifications will be discussed in relation to self-organizing systems. . . . the transformations and classifications suggested by a highly organized natural system, namely, the eye, will be reviewed first. . . . the way it works has relevance also for self-organizing systems." (p. 190)

I-16. MacArthur, Robert, "Fluctuations of Animal Populations, and a Measure of Community Stability," General Systems, Vol. 3, 1958, pp. 148-151.

Graph and information theory are integrated to provide a basis for discussing the relationship between efficiency and stability of a species in diverse ecological settings.

I-17. Mackay, D. M., "Towards an Information-Plan Model of Human Behavior," British Journal of Psychology, Vol. 47, 1956, pp. 30-43.

"This paper is concerned with the behavior possible in an information-flow system intended explicity as a hypothetical model for comparison with the human information-handling system." (p. 31)

I-18. Margalef, D. Ramon, "Information Theory in Ecology," General Systems, Vol. 3, 1958, pp. 36-71.

"Information theory describes the evolution of structured systems, divisible into elements qualitatively different, into states representing a greater degree of organization, in the individual as well as in the race and in the biosphere." (p. 69)

I-19. Mesarovic, Mihajlo D., "On Self-Organizational Systems," Self-Organizing Systems, ed. by Marshall C. Yovits, George T. Jacobi, and Gordon D. Goldstein, Washington, D. C.: Spartan Books, 1962, pp. 9-36.

"In order to put the development of self-organizing systems on a sound basis, it is necessary to define them in terms of the activities or behavior of the given general system and not in terms of the specifics of the system under consideration." (p. 9) "Without further investigations in the actual case, it is not possible now to decide which of the communication structural patterns is preferable." (p. 21)

I-20. Miller, James G., "Information Input Overload," Self-Organizing Systems, ed. by Marshall C. Yovits, George T. Jacobi, and Gordon D. Goldstein, Washington, D. C.: Spartan Books, 1962, pp. 61-78.

". . . When input information in bits per second is increased [in living systems] the output at first follows the input more or less as a linear function, then levels off at channel capacity and finally falls off toward zero." (p. 76) ". . . the more components there are in an information processing system, the lower is its channel capacity." (p. 77)

I-21. Pask, Gordon, "Interaction Between a Group of Subjects and an Adaptive Automaton to Produce a Self-Organizing System for Decision Making," Self-Organizing Systems, ed. by Marshall C. Yovits, George T. Jacobi, and Gordon D. Goldstein, Washington, D. C.: Spartan Books, 1962, pp. 283-311.

"My contention is that when a group is optimally organized, for inductive problem solving, the information structures that describe its state become a self-organizing system." (p. 284)

I-22. Pringle, J. W. S., "On the Parallel Between Learning and Evolution," General Systems, Vol. 1, 1956, pp. 90-110.

"The characteristic of living systems which distinguishes them most clearly from the non-living is their . . . evolution from less to more complex states of organization. . . . The key to its meaning [the meaning of the concept of complexity] comes from recent studies of the nature of "information" which have developed from problems of communications engineering." (p. 90)

I-23. Quastler, Henry, "Information Theory Terms and Their Psychological Correlates," Information Theory in Psychology, ed. by Henry Quastler, Glencoe, Illinois: The Free Press, 1955, pp. 143-171.

"A system is an organized whole made up of interrelated parts. . . . If two parts are interrelated in any fashion, then knowledge of the state of one must imply some information about the state of the other. . . . Accordingly, information measures can be used to evaluate any kind of organization." (pp. 159-160)

I-24. Quastler, Henry, The Emergence of Biological Organization, New Haven, Connecticut: Yale University Press, 1964.

"For individual components of biological systems the problem of organization is one of specification or information content. With pairs of components different problems arise relating to function, information transmission, action and interaction of information. . ." (p. ix)

I-25. Rapoport, A. and Horvath, W. J., "Thoughts on Organization Theory and a Review of Two Conferences," General Systems, Vol. 4, 1959, pp. 87-93.

". . . cybernetics is a dynamics superimposed on topology. It seems likely that these two disciplines will be at the foundation of that branch of science which deals with "organized complexity," i.e., organization theory." (p. 90)

1-26. Rashevsky, N., "Some Bio-Sociological Aspects of the Mathematical Theory of Communication," *Bulletin of Mathematical Biophysics*, Vol. 12, 1950, pp. 359-378.

"In the first part of the paper a general discussion of the transmission of information through neural chains is given. . . . In the second part transmission of information through "social chains" is discussed under certain special assumptions." (p. 359)

1-27. Rashevsky, N., "A Note on the Theory of Communication Through Social Channels," *Bulletin of Mathematical Biophysics*, Vol. 13, 1951, pp. 139-146.

"In a previous paper [1-25] a theory of transmission of information through a chain of individuals was developed. . . . In the present paper the theory is generalized . . ." (p. 139)

1-28. Rashevsky, N., "Life, Information Theory, and Topology," *Bulletin of Mathematical Biophysics*, Vol. 19, 1955, pp. 229-235.

"A study of the relations between the topological properties of graphs and their information content is suggested, and several theorems are demonstrated." (p. 229)

1-29. Rashevsky, N., "Life, Information Theory, Probability, and Physics," *Bulletin of Mathematical Biophysics*, Vol. 22, 1960, pp. 351-364.

The information which an organism must possess to replicate itself is used to infer the unlikeliness of a spontaneous generation by pure chance during the lifetime of the earth. "Dynamic factors, which . . . reduce . . . the information content, must play a role in the genesis of life on earth." (p. 351)

1-30. Raymond, Richard C., "Communication, Entropy, and Life," *American Scientist*, Vol. 38, 1950, pp. 273-278.

"It is the purpose of this paper to suggest that the entropy of a system may be defined quite generally as the sum of the positive thermodynamic entropy which the constituents of the system would have at thermodynamic equilibrium and a negative term proportional to the information necessary to build the actual system from its equilibrium state." (p. 273)

I-31. Rothstein, Jerome, "Information, Organization and Systems," IEEE Transactions on Information Theory, Vol. IT-4, 1954, pp. 64-66.

"The object of this paper is to develop and apply a mathematical concept of organization and of systems. It is very closely related to the information concept and provides the link whereby the theorems of communication theory become generalized and applicable to systems in general. Brief applications are given to system reliability, the significance of organization theory for circuit design, and production and quality control for a systems viewpoint." (p. 64)

I-32. Shimbel, A., "Application of Matrix Algebra to Communication Nets," Bulletin of Mathematical Biophysics, Vol. 13, 1951, pp. 165-178.

"A "generic" problem [a system containing n objects for which is associated with every ordered pair of elements the affirmation or negation of k relations] amenable to matrix algebraic treatment is outlined. Several examples are given and one, a communication system, is studied in some detail." (p. 165)

I-33. Siegel, S., "Theoretical Models of Choice and Strategy Behavior: Stable State Behavior in the Two-Choice Uncertain Outcome Situation," Psychometrika, Vol. 24, 1959, pp. 303-316.

"A theoretical approach to the understanding of human behavior in uncertain outcome situations is suggested, an approach which draws upon utility theory, decision-making theory, and statistical association theory." (p. 303) Three models are presented, the third of which interrelates Shannon's information theory with utility theory.

I-34. Soest, J. L. van, "A Contribution of Information Theory to Sociology," Synthese, Vol. 9, 1954, pp. 265-273.

". . . I have tried to find some parallelism between the degree of organization and of disorganization in physical and in sociological systems." (p. 265)

I-35. Stahl, Walter R., "Dimensional Analysis in Mathematical Biology," Bulletin of Mathematical Biophysics, Vol. 23, 1961, pp. 355-376.

Dimensional analysis clarifies the relationship of entropy to information and points the way to the formulation of many new functions based on the H-function.

I-36. Tribus, Myron, "Information Theory as the Basis for Thermostatics and Thermodynamics," General Systems, Vol. 6, 1961, pp. 127-138.

"By the use of Jaynes' proposed formalism for statistical inference it is possible to use the results of information theory and six essentially transparent axioms to derive all of classical thermodynamics and to review irreversible thermodynamics in a clearer light." (p. 127)

I-37. Trucco, Ernesto, "On the Information Content of Graphs: Compound Symbols; Different States for Each Point," Bulletin of Mathematical Biophysics, Vol. 18, 1956, pp. 237-253.

"The idea of assigning an information content to a graph is extended to . . . (a) Combination of set of topologically equivalent points . . . used as symbols; (b) Points of the graph . . . in different states." (p. 237)

I-38. Valentinuzzi, Maximo and Valentinuzzi, M. Eugenio, "Information Content of Chemical Structures and Some Possible Biological Application," Bulletin of Mathematical Biophysics, Vol. 25, 1963, pp. 11-27.

"Information content, theoretical physical entropy, real physical entropy, informational entropy or negentropy, entropic information or neginformation, heat amount, as well as relationships between these system parameters are defined and used." (p. 11)

Of Graph Theory

The literature indicates that there has been no explicit relating of graph theory to general systems theory per se. The relating has been with biological systems, as carried out mainly by a group of mathematical biologists at the University of Chicago, and with behavioral systems, as carried out mainly by a group at the Research Center for Group Dynamics at the University of Michigan. In other words, Rashevsky and Rosen of Chicago and Harary and Cartwright of Michigan are the outstanding contributors.

It is important to note that relatings in which graph theory is characterized in terms of other theories also are included. For example, Katz characterizes graph theory in terms of matrix theory (G-30, G-31), while Rosen does so in terms of categorical algebra (G-67 through G-75).

G-1. Bavelas, A., "A Mathematical Model for Group Structures," *Applied Anthropology*, Vol. 7, 1948, pp. 16-30.

A graph theoretic representation of a group is utilized to define the distance between two cells and centrality.

G-2. Bavelas, A., "Communication Patterns in Task-Oriented Groups," *The Policy Sciences*, ed. by D. Lerner and H. D. Lasswell, Stanford, Calif.: Stanford University Press, 1951, pp. 193-202.

A description of the activity and general direction of work on communication patterns in task-oriented groups is presented.

G-3. Cartwright, Dorwin, "The Potential Contribution of Graph Theory to Organizational Theory," *Modern Organization Theory: A Symposium of the Foundation for Research on Human Behavior*, ed. by Mason Haire, New York: John Wiley and Sons, 1959, pp. 254-274.

The usefulness of graph theory as a tool for the organizational theorist is indicated.

G-4. Davis, Robert L., "Structures of Dominance Relations," *Bulletin of Mathematical Biophysics*, Vol. 16, 1954, pp. 131-140.

The number of dominance structures for a society of n members as defined in G-34, G-35, G-49, and G-51 is determined.

G-5. Feather, N. T., "A Structural Balance Model of Communication Effects," *Psychological Review*, Vol. 71, 1964, pp. 291-313.

Cartwright and Harary's discussion of structural balance is considered for more complicated signed graphs and related to studies from different areas.

G-6. Festinger, L., "The Analysis of Sociograms Using Matrix Algebra," *Human Relations*, Vol. 2, 1949, pp. 153-158.

The advantages of matrix representation of group structure and some unsolved problems which arise from such a representation are discussed.

G-7. Ford, L. R., Jr. and Fulkerson, D. R., Flows in Networks, Princeton, New Jersey: Princeton University Press, 1962, 194 pp.

". . . linear graphs . . . relate . . . to the practically oriented subject of flows in networks." (p. vii)

G-8. French, John R. P., Jr., "A Formal Theory of Social Power," Psychological Review, Vol. 63, 1956, pp. 181-193.

Linear graph theory and matrix theory are used in formulating a theory of social power.

G-9. French, John R. P., Jr. and Raven, Bertram, "The Bases of Social Power," Studies in Social Power, ed. by D. Cartwright, Ann Arbor, Michigan: Institute for Social Research, The University of Michigan, 1959, pp. 150-167.

On the basis of the graph theoretic approach to group interaction, types of social power are discussed.

G-10. Harary, Frank, "Structural Duality," Behavioral Science, Vol. 2, 1957, pp. 255-265.

Structural, existential, and directional duality provide extensions of graph theory.

G-11. Harary, Frank, "A Criterion for Unanimity in French's Theory of Social Power," Studies in Social Power, ed. by D. Cartwright, Ann Arbor, Michigan: Institute for Social Research, The University of Michigan, 1959, pp. 168-182.

A necessary and sufficient condition for the attainment of ultimate unanimity of opinions in a power structure is presented. This item relates to G-8.

G-12. Harary, Frank, "Graph Theoretic Methods in the Management Sciences," Management Science, Vol. 5, 1959, pp. 387-403.

The utility of graph theory for characterizing structure in the management sciences is shown.

G-13. Harary, Frank, "On the Measurement of Structural Balance," Behavioral Science, Vol. 4, 1959, pp. 316-323.

"This paper continues the exploration of the mathematical properties of signed graphs with special reference to specific assumptions about the evolution of human groups." (p. 316)

G-14. Harary, Frank, "Status and Contrastatus," Sociometry, Vol. 22, 1959, pp. 23-43.

"Our object is to propose a formula to measure the positional aspect of the status of a person in an organization or group, and investigate some of its ramifications." (p. 23)

G-15. Harary, Frank, "Who Eats Whom?" General Systems, Vol. 6, 1961, pp. 41-44.

The structure of an ecological system involving several animals is represented by a directed graph, and the status of each animal is calculated.

G-16. Harary, Frank, "A Graph Theoretic Approach to Similarity Relations," Psychometrika, Vol. 29, 1964, pp. 143-151.

". . . similarity relations are naturally expressible as graphs and . . . their concepts are subsumed within the framework of graph theory." (p. 143)

G-17. Harary, Frank and Cartwright, D., "Structural Balance: A Generalization of Heider's Theory," Psychological Review, Vol. 63, 1956, pp. 277-293.

Through the utilization of graph theory, F. Heider's concept of structural balance in small groups is explicated and generalized.

G-18. Harary, Frank and Norman, Robert Z., Graph Theory as a Mathematical Model in Social Science, Ann Arbor, Michigan: Institute for Social Research, The University of Michigan, 1953, 45 pp.

The content of graph theory and its relation to the social sciences are outlined.

G-19. Harary, Frank and Ross, I. C., "On the Determination of Redundancies in Sociometric Chains," Psychometrika, Vol. 17, 1952, pp. 195-208.

The number of redundant paths in communication networks is determined.

G-20. Harary, Frank and Ross, I. C., "The Number of Complete Cycles in a Communication Network," Journal of Social Psychology, Vol. 40, 1954, pp. 329-332.

"We wish to deduce a criterion for the existence of a path in a network such that a rumor . . . initiated by any member will return to him after having passed through each of the other persons exactly once." (p. 329)

G-21. Harary, Frank and Ross, I. C., "A Procedure for Clique Detection Using the Group Matrix," Sociometry, Vol. 20, 1957, pp. 205-215.

Festinger's results stated in G-6 are extended: cocliqual and unicliqual persons are characterized, and theorems concerning groups of k cliques are proved.

G-22. Harary, Frank and Ross, I. C., "A Description of Strengthening and Weakening Members of a Group," Sociometry, Vol. 22, 1959, pp. 139-147.

"This paper is one of a continuing series in which matrix analysis is used to identify various aspects of group structure." (p. 139)

G-23. Heise, G. A. and Miller, G. A., "Problem-solving by Small Groups Using Various Communication Nets," Journal of Abnormal and Social Psychology, Vol. 46, 1951, pp. 327-335.

"The present experiment is similar in many respects to Leavitt's [G-30], although the control of the situation is carried still further." (p. 327)

G-24. Hodgson, Anthony M., "The Solution of Technical Problems by Groups, A System Model of a Task-Group," Systematics, Vol. 2, 1964, pp. 1-46.

The purpose is to provide a systematic model of creative problem solving groups. Graphs are used to represent the structure of such a group with the vertices representing such determining factors of group performance as human purpose. Graphs are used also to represent various modes of group operation.

G-25. Hodgson, Anthony H., "The Solution of Technical Problems by Groups - 2," Systematics, Vol. 2, 1964, pp. 177-213.

"[Graphs are used] to construct a sufficiently generalized structural model to account for problem-solving in a number of fields. This model, together with the task-group model of Part One [G-24], is presented as a conceptual basis for creating new techniques of training professional workers in creative group problem-solving." (p. 177)

G-26. Hodgson, Anthony M., "The Solution of Technical Problems by Groups - 3," Systematics, Vol. 2, 1964, pp. 298-322.

"A comparison of the problem-solving model and various fields of work is given, followed by some observations on how problem-solving stages synchronise with task-group behaviour." (p. 298)

G-27. Iri, op. cit. in I-9.

G-28. Jamrich, John X., "Application of Matrices in the Analysis of Sociometric Data," Journal of Experimental Education, Vol. 28, 1960, pp. 249-252.

"The use of the characteristic column-vector [of a matrix] . . . affords a relatively simple procedure for determining a numerical value for the status of individuals in a group." (p. 252)

G-29. Karreman, op. cit. in I-13.

G-30. Katz, Leo, "An Application of Matrix Algebra to the Study of Human Relations within Organizations," Chapel Hill, North Carolina: Final Report under Project NR 042 031, University of North Carolina, 1950.

Various applications of matrix algebra in the representation of group structure and the explication of group properties are discussed.

G-31. Katz, Leo, "A New Status Index Derived from Sociometric Analysis," Psychometrika, Vol. 18, 1953, pp. 39-43.

Representation of group structure by matrices permits a more adequate definition of status ". . . which takes into account who chooses as well as how many choose" (p. 39).

G-32. Kemeny, John G. and Snell, J. Laurie, "Organization Theory Applications of Graph Theory," Mathematical Models in the Social Sciences, New York: Ginn and Company, New York, 1962, pp. 95-108.

A characterization of a balanced graph is obtained, a general measure of status is given for hierarchical graphs, and the notion of sameness of importance in a graph is explicated by Markov chain theory.

G-33. Kochen, Manfred, "Organized Systems with Discrete Information Transfer," General Systems, Vol. 2, 1957, pp. 30-47.

Graph theory is used to characterize certain component structures in systems with discrete information transfer.

G-34. Landau, H. G., "On Dominance Relations and the Structure of Animal Societies: I. Effect of Inherent Characteristics," Bulletin of Mathematical Biophysics, Vol. 13, 1951, pp. 1-19.

Matrix theory is used to represent the structure of a society and a probabilistic measure of degree of societal hierarchy is defined.

G-35. Landau, H. G., "On Dominance Relations and the Structure of Animal Societies: II. Some Effects of Possible Social Factors," Bulletin of Mathematical Biophysics, Vol. 13, 1951, pp. 245-262.

The theory introduced in G-34 is extended using Markov chain theory. It is concluded that social factors could account for discrepancies between theory and data reported in the earlier article.

G-36. Landau, H. G., "On Dominance Relations and the Structure of Animal Societies: III. The Condition for a Score Structure," Bulletin of Mathematical Biophysics, Vol. 15, 1953, pp. 143-148.

Score structure of a society with a dominance relation is characterized, and it is proven that there are members who dominate every other member directly or indirectly through a single intermediate member.

G-37. Landau, H. G., "The Distribution of Completion Times for Random Communication in a Task-Oriented Group," Bulletin of Mathematical Biophysics, Vol. 16, 1954, pp. 187-201.

Using a Markov chain model for communication nets, expected completion times are computed for various types of nets.

G-38. Leavitt, H. J., "Some Effects of Certain Communication Patterns on Group Performance," Journal of Abnormal and Social Psychology, Vol. 46, 1951, pp. 38-50.

"It was the purpose of this investigation to explore experimentally the relationship between the behavior of small groups and the patterns of communication in which the groups operate . . . [and] to consider the psychological conditions that are imposed upon group members by various communication patterns, and the effects of these conditions . . . ." (p. 38)

G-39. Leeman, C. P., "Patterns of Sociometric Choice in Small Groups: A Mathematical Model and Related Experimentation," Sociometry, Vol. 15, 1952, pp. 220-243.

"The question of what sociometric patterns are possible in a group of given size is discussed; and a mathematical model is presented, treating the likelihood of the several patterns, and the variation in pattern upon repeated sociometric choices." (p. 220)

G-40. Luce, R. D., "Connectivity and Generalized Cliques in Sociometric Group Structure," Psychometrika, Vol. 15, 1950, pp. 169-190.

". . . the number of elements in a group, the number of animetrics, and the degree of connectivity must supply certain inequalities." (This quotation is from Psychological Abstracts, Vol. 25, 1951, p. 699.)

G-41. Luce, R. D. and Perry, Albert D., "A Method of Matrix Analysis of Group Structure," Psychometrika, Vol. 14, 1949, pp. 95-116.

"Matrix methods may be applied to the analysis of experimental data concerning group structure when these data indicate relationships which can be depicted by line diagrams . . . ." (p. 95) The clique structure of a group is analyzed using matrix theory.

G-42. MacArthur, op. cit. in I-16.

G-43. Maccia, Elizabeth Steiner, "An Educational Theory Model: Graph Theory," Construction of Educational Theory Models, by E. S. Maccia, G. S. Maccia, and Robert E. Jewett, Washington, D. C.: Office of Education, U. S. Department of Health, Education, and Welfare, Cooperative Research Project No. 1632, 1963, pp. 101-138.

". . . graph theory [is shown to be] utilizable in formulating characterizations of relations of persons in groups within the schools." (p. 101)

G-44. Martinez, op. cit. in S-2.

G-45. Morrissette, Julian O., "An Experimental Study of the Theory of Structural Balance," Human Relations, Vol. 11, 1958, pp. 239-254.

"The present paper directs its attention to the formalization of Heider's theory advanced by Cartwright and Harary [G-17] and to the empirical testing of the consequences following from this formalization." (p. 239)

G-46. Oeser, O. A. and Harary, Frank, "A Mathematical Model for Structural Role Theory, I," Human Relations, Vol. 15, 1962, pp. 89-122.

Graph theory is used ". . . to consider role . . . as a structure concept and to formulate some definitions" (p. 90).

G-47. Oeser, O. A. and Harary, Frank, "A Mathematical Model for Structural Role Theory, II," Human Relations, Vol. 17, 1964, pp. 3-17.

"The purpose of this paper is to develop further some general concepts and theoretical considerations about the structure of role systems which were discussed in Part I [G-46]." (p. 3)

G-48. Ramanajachargulu, C., "Analysis of Preferential Experiments," Psychometrika, Vol. 29, 1964, pp. 257-261.

Using graph theory, the person with the most power to influence and the least power to be influenced is located for a 5-person group.

G-49. Rapoport, Anatol, "Outline of a Probabilistic Approach to Animal Sociology: I," Bulletin of Mathematical Biophysics, Vol. 11, 1949, pp. 183-196.

Graph theory is used to characterize structure in a partially probabilistic theory of animal societies in which an antisymmetric relation exists between members of the society.

G-50. Rapoport, Anatol, "Outline of a Probabilistic Approach to Animal Sociology: II," Bulletin of Mathematical Biophysics, Vol. 11, 1949, pp. 273-281.

"Under certain assumptions . . . the probability distribution for all possible structures of a society . . . approaches a limit independent of the initial probability distribution."

G-51. Rapoport, Anatol, "Outline of a Probabilistic Approach to Animal Sociology: III," Bulletin of Mathematical Biophysics, Vol. 12, 1950, pp. 1-17.

"The probabilities of the emergence of two kinds of social structure in a 3-bird flock . . . are deduced under the assumption of certain biases acting on the social dynamics of the flock." (p. 7)

G-52. Rapoport, Anatol, "Contribution to the Theory of Random and Biased Nets," Bulletin of Mathematical Biophysics, Vol. 19, 1957, pp. 257-277.

Graph theory is used as one of the bases for a discussion of the statistical structure of random and biased nets.

G-53. Rashevsky, N., "Topology and Life: In Search of General Mathematical Principles in Biology and Sociology," Bulletin of Mathematical Biophysics, Vol. 16, 1954, pp. 317-348.

The graph of the primordal organism which represents the relations among different biological functions is postulated. Transformation rules for the graph are posited such that from this graph could be derived the graphs of all higher organisms.

G-54. Rashevsky, N., "Some Theorems in Topology and a Possible Biological Implication," Bulletin of Mathematical Biophysics, Vol. 17, 1955, pp. 111-126.

This paper investigates the properties of transformations of graphs defined in a previous paper (G-53). ". . . considerations suggest the possibility of deriving some general biological laws from the consideration of the properties of the transformation only . . . ." (p. 111)

G-55. Rashevsky, N., "Some Remarks on Topological Biology," Bulletin of Mathematical Biophysics, Vol. 17, 1955, pp. 207-218.

The choice of a particular primordal graph and transformation (G-53) defines an abstract biology. Two theorems lead to the conclusion that the higher the organism the more adaptable. Inadequacies in the theory are noted, and suggestions made.

G-56. Rashevsky, op. cit. in I-28.

G-57. Rashevsky, N., "The Geometrization of Biology," Bulletin of Mathematical Biophysics, Vol. 18, 1956, pp. 31-56.

"[To geometrize biology] . . . we must find geometric structures or spaces, in which different geometric properties stand to each other in the same formal logical relation, as the different concepts of biology stand to each other." (p. 31) Several example spaces are introduced, and verifiable predictions are made.

G-58. Rashevsky, N., "Contributions to Topological Biology: Some Considerations on the Primordal Graph and on Some Possible Transformations," Bulletin of Mathematical Biophysics, Vol. 18, 1956, pp. 113-128.

This paper is a continuation of the one cited in G-53. A different primordal graph and different transformations are proposed, and implications are discussed.

G-59. Rashevsky, N., "What Type of Empirically Verifiable Predictions Can Topological Biology Make?" Bulletin of Mathematical Biophysics, Vol. 18, 1956, pp. 173-188.

Considerations of topological biology, e.g. the total number of possible organisms, are of the type which would not be considered in the usual metric approach to biology.

G-60. Rashevsky, N., "A Note on the Geometrization of Biology," Bulletin of Mathematical Biophysics, Vol. 19, 1957, pp. 201-204.

Dependence of studied topological spaces on the type of space in which they are embedded reflects some aspects of the dependence of the organism on its environment.

G-61. Rashevsky, N., "Remark on an Interesting Problem in Topological Biology," Bulletin of Mathematical Biophysics, Vol. 19, 1957, pp. 205-209.

"The possibility of several homotopic classes of mappings of the graph of an organism onto the primordal graph . . . is considered." (p. 205)

G-62. Rashevsky, op. cit. in S-4.

G-63. Rashevsky, op. cit. in S-5.

G-64. Rashevsky, op. cit. in S-6.

G-65. Rashevsky, *op. cit.* in S-7.

G-66. Rashevsky, *op. cit.* in S-8.

G-67. Rosen, *op. cit.* in S-9.

G-68. Rosen, *op. cit.* in S-10.

G-69. Rosen, *op. cit.* in S-11.

G-70. Rosen, *op. cit.* in S-12.

G-71. Rosen, *op. cit.* in S-13.

G-72. Rosen, *op. cit.* in S-14.

G-73. Rosen, *op. cit.* in S-15.

G-74. Rosen, *op. cit.* in S-16.

G-75. Rosen, *op. cit.* in S-17.

G-76. Rosenblatt, David, "On Some Aspects of Models of Complex Behavioral Systems," *Information and Decision Processes*, ed. by Robert A. Machol, New York: McGraw-Hill Book Co., 1960, pp. 62-86.

"In this paper we propose to treat some . . . aspects of certain models of complex behavioral systems . . . [especially] the abstract concepts of *balance*, *closure*, and *interaction*." (pp. 62-63)

G-77. Shimbel, Alfonso, "Applications of Matrix Algebra to Communication Nets," *Bulletin of Mathematical Biophysics*, Vol. 13, 1951, pp. 165-178.

The structure and status matrix of a communication system is defined as a function of time, and certain theorems relating to the solution of a group problem are derived.

G-78. Shimbel, Alfonso, "Structural Parameters of Communication Networks," *Bulletin of Mathematical Biophysics*, Vol. 15, 1953, pp. 501-507.

"Certain parameters are defined which roughly characterize the internal structure of networks, . . . the *dispersion* . . . gives an indication of the "compactness" of the internal structure." (p. 501)

G-79. Solomonoff, Ray, "An Exact Method for the Computation of the Connectivity of Random Nets," *Bulletin of Mathematical Biophysics*, Vol. 14, 1952, pp. 153-157.

"The problem of finding the "weak connectivity" of a random net is reduced to one involving a Markov process." (p. 153)

G-80. Solomonoff, Ray and Rapoport, Anatol, "Connectivity of Random Nets," *Bulletin of Mathematical Biophysics*, Vol. 13, 1951, pp. 107-117.

Weak connectivity, i.e. the expected number of neurons to which there exists paths from an arbitrary neuron of a random net, is defined and is an indicator of maximum expected spread of an epidemic under certain conditions.

G-81. Trucco, *op. cit.* in I-37.

G-82. Trucco, Ernesto, "Topological Biology: A Note on Rashevsky's Transformation T," *Bulletin of Mathematical Biophysics*, Vol. 19, 1957, pp. 19-21.

A theorem proved in an earlier article by Rashevsky (G-59) is generalized.

G-83. Trucco, Ernesto, "Note on a Combinatorial Problem," *Bulletin of Mathematical Biophysics*, Vol. 19, 1957, pp. 309-336.

The problem of finding the total number of graphs that can be obtained from the biotopological transformation (T(1)x) for a given value of the parameter n is partially solved.

G-84. Wei, Telson, "On Matrices of Neural Nets," *Bulletin of Mathematical Biophysics*, Vol. 10, 1948, pp. 63-67.

"The structure of a . . . neural net is represented here by several matrices." (p. 63)

Of Set Theory

In an earlier work[3] a system was taken to be a set (where 'set' was used in a common sense way) of entities together with their properties and the relationships between the entities. Substituting 'objects' for 'entities' and 'attributes' for properties', Hall and Fagen's definition[4] results. Neither of these definitions nor von Bertalanffy's[5] nor Grinker's[6] nor Cherry's[7] were found sufficiently explicit. Set theory provides the basic means for eliminating this inadequacy.

A system is taken to be a group of at least two components with at least one affect relation and with information. Utilizing set theory, the group of at least two components becomes a set of at least two elements which form a sequence. The conditions, too, are given meaning ultimately in terms of set theory. A relation between components of the system, an affect relation, is given meaning through digraph theory which is based on set theory. Through digraph theory, the group of a system becomes a set of points and an affect relation a set of directed lines. Not only is set used, but also the set theoretic definition of 'function'. An affect relation is a mapping of the group into itself. Through information theory, information of a system becomes a characterization of system occurrences at categories in a classification. System occurrences may be with respect to either system components or system affect relations or both. Since a classification is a set of categories, set theory also is basic to information theory.

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[3] G. S. Maccia, "An Educational Theory Model: General Systems Theory," Construction of Educational Theory Models, Washington, D. C.: Cooperative Research Project No. 1632, Office of Education, U. S. Department of Health, Education, and Welfare, 1963, p. 140.

[4] "Definition of System," General Systems, Vol. 1, 1956, p. 18.

[5] "General Systems Theory," General Systems, Vol. 1, 1956, p. 3.

[6] "Summary," Towards a Unified Theory of Behavior, ed. by Grinker, New York: Basic Books, 1956, p. 310.

[7] On Human Communication, Cambridge, Mass.: Technology Press of Massachusetts Institute of Technology, 1957, p. 307.

Properties of a system are not part of the definition of 'a system'. Rather properties are subsets of systems which are sorted out from the set of all systems, because they have conditions on them over and above the conditions which make them a system. For example, a subset of systems with environmentness are those systems from the set of all systems which have the added condition of a negasystem with at least two components and at least one affect relation and selective information. Selective information involves uncertainty with respect to the occurrences of either components or affect relations or both. The property of system environmentness is called also 'toputness'. Thus, all systems do not necessarily have an environment, but whenever there is a system there is a negasystem.

The set characterization, complement, gives meaning to a negasystem. Within whatever universe of discourse is selected, the components selected for consideration, the components which do not belong to the system are the negasystem. See Figure 1 on page 99.

Negasystems, too, can have properties. To illustrate: negasystems can have the property of fromputness or environmentness, if they have the condition of a system with selective information. Negasystems with this added condition of selectivity of the information on the systems of which they are complements are subsets of the set of all negasystems. Thus, all negasystems do not have an environment, but whenever there is a negasystem there is a system.

Both the state of a system and of a negasystem are given meaning as sets in which the elements are conditions of a given system or negasystem at a certain time. The properties too can have states. The set theoretic characterization, function, is employed in setting forth what a system property state and a negasystem property state are. That function which has respective values in its image space is involved, for a property state is a value of a property at a given time.

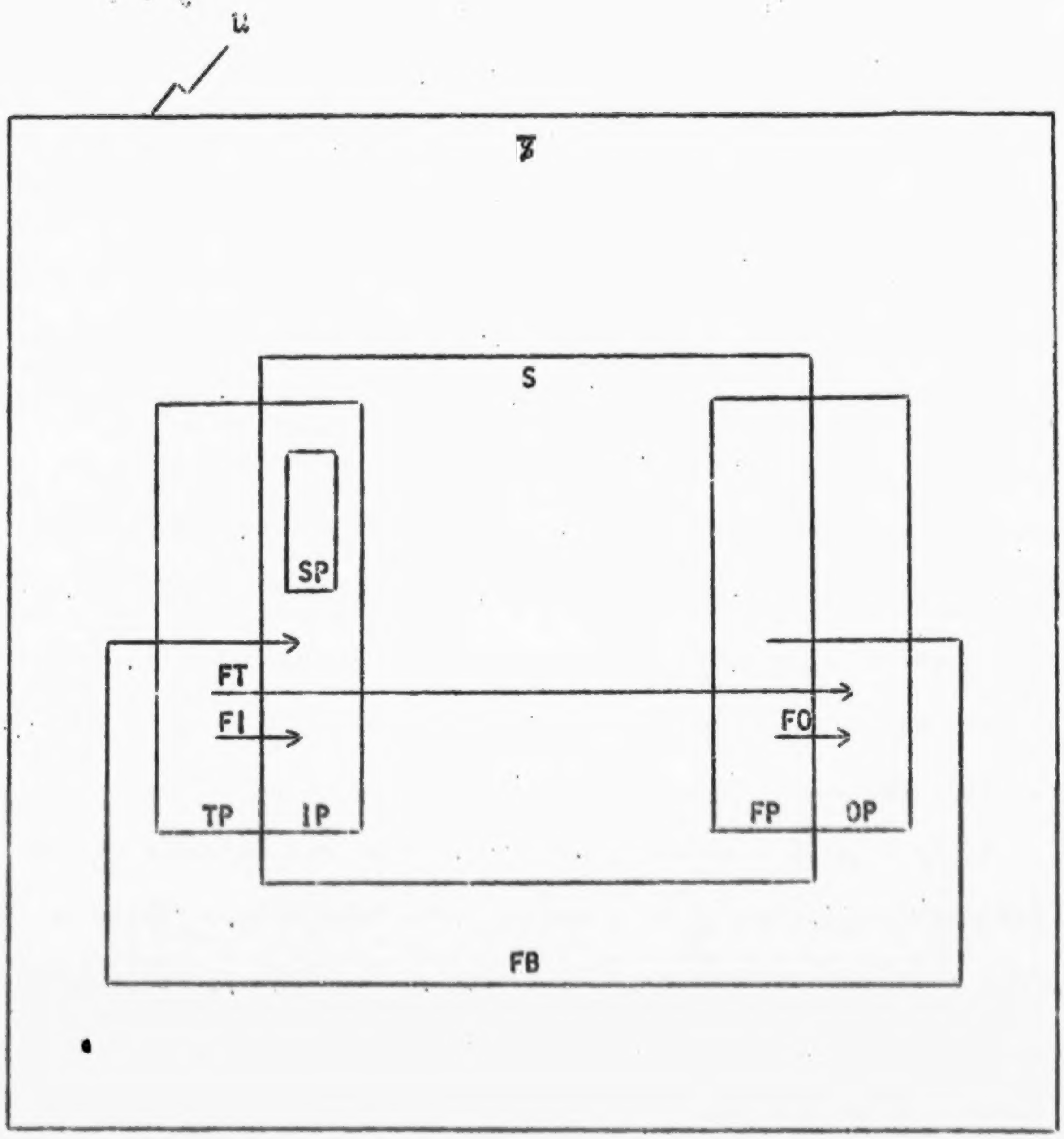
The conditions which systems must satisfy to be characterized in terms of certain properties depend also upon set theory. Explicit use of set theory is exemplified in the conditions with regard to sizeness and homomorphismness. In the former the set theoretic characterization, cardinality, is explicit, while in the latter homomorphic mapping is. Implicit use occurs throughout the conditions, for both information theory and digraph theory are based upon set theory.

Of Information Theory

As already stated, every system has information in the sense that occurrences of its components or affect relations or both can be classified according to categories. A system, in other words, can be characterized. The added condition of selectivity of the information, i.e. uncertainty of occurrences at the categories, is required to develop information properties of systems and negasystems and of their states. Uncertainty of occurrences is explicated in terms of a probability distribution. For example, if there is uncertainty with respect to an occurrence of a system component at a category of a classification of the system components, then the probability at the category can be neither 1 or 0 but must be less than 1 or greater than 0. Consequently, there must be at least one alternative category for the occurrence of the component, since the sum of the probabilities must be equal to 1.

Figure 1 on the following page summarizes and illustrates the basic information properties of a system and a negasystem. Those of the system are toputness, inputness, storeputness, feedinness, feedoutness, feedthroughness, and feedbackness, while those of the negasystem are fromputness and outputness.

Only the condition of selectivity is required to give meaning to toputness, inputness, fromputness, and outputness. Both toputness and outputness involve selective information on a negasystem whereas fromputness and inputness involve selective information on a system. Nevertheless toputness can be sorted out from outputness, and fromputness



'$\mathfrak{U}$' stands for universe of discourse
'$\bar{S}$' stands for system
'$\not{S}$' stands for negasystem
'SP' stands for storeputness
'FT' stands for feedthroughness
'FI' stands for feedinness

'TP' stands for toputness
'IP' stands for inputness
'FO' stands for feedoutness
'FP' stands for fromputness
'OP' stands for outputness
'FB' stands for feedbackness

Figure 1

from inputness. Toputness is a system property, a system's environment or the selective information on a negasystem available to a system, but outputness is a negasystem property, its selective information. Likewise, fromputness is a negasystem property, a negasystem's environment or the selective information on a system available to a negasystem, but inputness is a system's property, its selective information.

The other basic information properties require conditions over and above that of selectivity. Storeputness of a system requires the selective information to be conditional, since storeputness is system selective information which results when one takes into account the dependency of system selective information upon that available to a negasystem. In other words, storeputness is the dependency of inputness upon fromputness. Feedinness, feedoutness, feedthroughness and feedbackness are properties in which there is a flow of selective information, a transmission of selective information. Conditions, hence, of selective information separated by time intervals and of sharing of selective information are requirements. To illustrate: feedinness, which is a transmission of selective information from a negasystem to a system, involves selective information on a negasystem available to a system, toputness, at a time just preceding a time at which some or all of that selective is on that system. In other words, feedinness is the shared information between toputness and inputness, where the toputness is at a time just prior to the inputness.

Of Graph Theory

As indicated earlier, through digraph theory a system group becomes a set of points and system affect relations become sets of directed lines. Within such a context, digraph properties of a system result when certain conditions are placed on its affect relations or its group.

Complete connectionness, strongness, unilaternalness, weakness, and disconnectionness illustrate digraph properties of a system arising from certain conditions placed on its affect relations. The conditions involved are directedness and directedness with the added condition of direct, as well as conditions as to the components contained in the affect relations. When an affect relation has directedness, the set of directed lines is such that one can trace a path with the direction of the lines from one or more components to one or more other components. In other words, there are channels between components. If the condition, direct, is added to directedness, then the channels do not run through other components; while if the condition, indirect, is added, the channels do so run. When an affect relation does not have directedness, there are no specified channels. The path between the components could just as well be traced against as with the direction of the lines.

Complete connectionness is a property in which affect relations are direct directed ones and in which every two components are contained. There are direct channels back and forth between every two components.

Strong systems do not have the property of complete connectionness, yet the affect relations are directed ones and every two components are contained in them. That is to say, there are channels back and forth between every two components, but they are not direct ones.

In unilateral systems the properties of complete connectionness and strongness do not obtain, yet again the affect relations are directed ones and every two components are contained in them. What is lacking is mutuality of the channels. There are only one-way channels.

Systems with the property of weakness lack the properties of complete connectionness, strongness, and unilateralness. The affect relations do not have the condition directedness placed on them and, moreover, directions are not specified so that there are channels. Nonetheless, every two components are contained in the affect relations.

Finally, disconnected systems have none of the above properties. Some of the components are not contained in the affect relations of a disconnected system.

Passive dependentness, active dependentness, independentness, and interdependentness exemplify digraph properties of a system due to conditions on the group. The conditions on the group have to do with the group component containment in affect relations. In passive dependentness, components are so contained that channels only go to the components; in active dependentness, channels only go from them; in independentness, channels do not go either to or from them; and, finally, in interdependentness channels go to and from them.

Comparison of Relations

In the literature and in the model both set theory and information theory are related to general systems[8] theory. Graph theory, however, is related to a system in general only in the model. In the literature graph theory is related only to kinds of systems.

The literature also reveals that there are no attempts to relate all three theories--set, information, and graph--to general systems theory as is done in the model. Two of the three theories--information and graph--are so related as indicated by the cross-referencing of items in the literature (e.g. I-9). Two of the three theories are related, moreover, only to kinds of systems: for example, set and graph theories to biological systems as in Rosen's work (S-9 through S-17), and information and graph theories to chemical systems as in I-13.

In S-1 and S-3 set theory is related to a general system. Both relatings differ from that in the model. For Ashby (S-1) a system is a set in which the elements are states, while for Mesarovic (S-3) a system is a set in which the elements are values. In the model, a system is a set in which the elements are components, while values only enter into property states of a system and states are sets of conditions of a system at a given time. Descriptive parameters are not taken as primitive in the model, since theory is taken to be essential to the demarcation of such parameters.[9]

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[8] As is common in the literature, the plural of 'system' is used. It would make more sense not to because "General has the same meaning as the s" (W. Ross Ashby, "General Comment," Society for General Systems Research, December, 1964, p. 3.)

That selective information theory can be related to the organization of a general system is indicated in some items of the literature (e.g. 1-3 and 1-8). Furthermore, explication as well as indication occurs, such as that of a self-organizing system by von Foerster in 1-8. In none of the items, however, is there a comprehensive explication of information theoretic properties of a general system as in the model. Moreover, there is uniqueness in this comprehensive explication, e.g. in the treatment of the environment of a system by means of toput as distinct from input and of the system as environment by means of fromput as distinct from output.

With respect to graph theory, not only does the model offer the only explicit relating of graph theory to a general system, but also the relating is done differently than in the items of the literature in which graph theory is related to a kind of system. The difference arises from the emphasis on the power of the graph theoretic approach to characterize structure rather than on extant characterizations of structure in terms of graph theory. Consequently, through basic definitions of 'affect relation', 'directed affect relation', 'direct directed affect relation', and 'indirect directed affect relation', graph theory becomes a powerful tool for a comprehensive explication of graph theoretic properties of a system.

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[9] For justification of this assertion see E. S. Maccia, "Ways of Inquiring," Construction of Educational Theory Models, op. cit., pp. 1-13.

CHAPTER VI

USE IN CONSTRUCTING EDUCATIONAL THEORY

Use in Literature

Annotated Bibliography

In order to furnish a basis for discussing the use of set theory or information theory or graph theory or any combination thereof, an annotated bibliography is presented.[1]

1. Abdel-Halim, Ahmed El-Mahdi, "An Intersystem Model for Curriculum Theory and Practice," Doctoral Dissertation, The Ohio State University, 1965.

   Curriculum is considered as a system of which three subsystems--content, psychological processes, and instructional setting--are identified and explicated. Also there is an attempt to establish four methodological hypotheses: "1. If curriculum is to be conceived as a system, then its main features can be identified and represented in a model. 2. If curriculum is to be treated as an intersystem model, then the patterns of relationship among its subsystems can be shown. 3. If curriculum is conceived as intersystem, the role or the roles played by each subsystem can be explained in its relative autonomy, and in its mutual reaction to other subsystems. 4. If curriculum is to be handled as a system, the sources of imbalance within the system due to internal and external change can be accounted for" (pp. 21-22).

2. Gordon, Ira J., "Developments in Human Behavior," Educational Theory, Vol. 8, 1958, pp. 259-268.

   Some properties of an open energy system are considered, and their relevance for education is discussed. For example: "We are saying that the individual is always active, is in constant transaction with the environment, is always moving in the direction of increased complexity, and acts to order himself and his environment" (p. 262), "We have said that information is essential to the system. It takes information to maintain organization. There must be input" (p. 265), and "knowledge of growth processes leads us to the recognition that one's physical organism sets limits on what will be perceived, experienced, turned into information, and therefore "learned"" (p. 266).

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[1] The items are numbered in order to facilitate reference to them in the discussion of the state of usage.

3. Griffiths, Daniel, "Some Assumptions Underlying the Use of Models in Research," Educational Research: New Perspectives, ed. by J. A. Culbertson and S. P. Hencley, Danville, Illinois: The Interstate Printers and Publishers, Inc., 1963, pp. 121-140.

   General systems theory is used for a theory of administrative performance. 'A system' is defined as a complex of elements in mutual interaction. Two kinds of systems are noted, open and closed, and seven properties of open systems are cited: inputs and outputs, steady states, self-regulation, equifinality, sub-systems, feedback, and progressive segregation. From these properties two propositions are developed: "The steady state of an administrative performance system is maintained by a decision process in which satisfactory alternatives are selected rather than optimal alternatives" (p. 137), and "Administrative systems respond to continuously increasing stress first by a lag in response, then by an over-compensating response, and finally by a catastrophic collapse of the system" (p. 138). Evidence supporting these two propositions is presented and discussed.

4. Henderson, Kenneth B., "A Logical Model for Conceptualizing and Other Related Activities," Educational Theory, Vol. 13, 1963, pp. 277-284.

   Set theory is used to explicate conceptualizing. Conceptualizing is considered as a 4-tuple relation which includes a term, a person, a context and a meaning. Since there is an explicit mention of the classroom, set theory could be considered as used in developing educational theory.

5. Husek, T. R., "Message Certainty and Information Destruction: Variables in the Study of the Communication of Information," California Journal of Educational Research, Vol. 15, 1964, pp. 184-189.

   "This paper describes an experiment on two variables related to the communication of information. . . . the results obtained have importance for the field of education." (p. 184) Since the experiment is obviously in the context of selective information theory, the construction of an educational theory in such a context is implied.

6. Kopstein, Felix F., "Methodological Considerations," Chapter of the ETS Report on the Pennsylvania Quality Education Project, Mimeographed Draft No. 2, May, 1965.

   General systems theory and system analysis are related in devising an evaluation procedure for measuring objectively the adequacy and efficiency of the educational programs of the public schools of the state of Pennsylvania. 'System' is defined as events taken over time among which there exist unidirectional, probabilistic relationships such that event p is necessary but not sufficient for event q to occur, although p implies q with a probability greater than zero. The properties--states, input, output, feedback, subsystems--are discussed within the framework of system analysis, so that specific quantitative measures can be associated with these properties in such a way that measures of adequacy and efficiency can be determined.

7. Maccia, Elizabeth Steiner, "An Educational Theory Model: Graph Theory," Construction of Educational Theory Models, by E. S. Maccia, G. S. Maccia, and Robert E. Jewett, Washington, D. C.: Office of Education, Department of Health, Education, and Welfare, Cooperative Research Project No. 1632, 1963, pp. 101-138.

   ". . . graph theory [is shown to be] utilizable in formulating characterizations of relations of persons in groups within the schools." (p. 101) The results are not only the educational theory model, but the tentative educational theory as well. Furthermore, some relevance and fruitfulness of the theory is indicated.

8. Maccia, Elizabeth Steiner, "An Educational Theory Model: Information Theory," Construction of Educational Theory Models, op. cit., pp. 298-334.

   Portions of information theory are presented and used as a model for constructing an educational theory in which the teaching-learning process becomes a problem solving or inquiry process. Studies which support and are suggested by the educational theory are cited.

9. Maccia, Elizabeth Steiner, "Instruction as Influence toward Rule-Governed Behavior," Theories of Instruction, ed. by James B. Macdonald and Robert R. Leeper, Washington, D. C.: Association for Supervision and Curriculum Development, 1965, pp. 88-99.

   On the basis of 7 and 8, a scientific theory is presented in which ". . . instruction is viewed as influence toward rule-governed behavior" (p. 91). The influence dimension of the theory is primarily concerned ". . . with structure variables which are descriptions of the internal behavior of the classroom group members" (p. 91). Digraph theory is used to represent influence structures, and the relation of these structures to other variables is indicated. The teacher, via five possible motivational bases which establish influence relations, builds up the cognitive structures of the student. The analysis of the structure of any discipline, therefore, is a prerequisite for its being taught. The rule-governed behavior dimension of the theory views the student as a problem-solver. The solving of problems is accomplished through cognitive structures (rules) which permit selection from sets of alternatives (problems), and hence the use of selective information theory is apparent.

10. Maccia, George S., "An Educational Theory Model: General Systems Theory," Cooperative Research Project No. 1632, *Construction of Educational Theory Models*, *op. cit*., pp. 139-172.

Terms and propositions from general systems theory are organized into a theory model. The theory model is utilized then to set forth a tentative educational theory in which the school is characterized as a system with certain properties and actions. Studies which support and are suggested by the educational theory are cited.

11. Macdonald, James B., "Curriculum Theory: Problems and a Prospectus," Madison, Wisconsin: The University of Wisconsin, Mimeographed Copy of a Paper Read at the Professors of Curriculum Meeting, Miami Beach, Florida, 1964, 17 pp.

Characterizations from general systems theory--system, input, output, feedback, adaptation, integration, tension channel, and goal achievement--are used as a perspective for generating some hypotheses of curriculum theory. Three hypotheses are cited. One hypothesis is as follows: "*That extensive curriculum change along lines associated with professional and/or behavioral input dimensions is dependent upon manipulation of impetus coming from a broad social context*" (p. 16). The presentation in this paper is primarily an illustrative discussion of the possibilities for using general systems theory as an heuristic source for curriculum theorizing.

12. Miller, James G., "The Information Explosion: Implications for Teaching," *Journal of the National Association of Women Deans and Counselors*, Vol. 27, 1964, pp. 54-59.

Studies on information processing in living systems when there is an overload have suggested several types of adjustment processes. It is suggested that education become more oriented to teaching the student these methods of information processing. Although Miller simply applies the theory directly to educational practice, nevertheless a full development of the suggestion would result in an educational theory undergirding the practice. Consequently, the paper is cited in this annotated bibliography.

13. Novak, Joseph D., "An Approach to the Interpretation and Measurement of Problem Solving Ability," *Science Education*, Vol. 45, 1961, pp. 122-131.

"A theory of problem solving behavior based on an interpretation of the role of stored information and acquired information in selection of "courses of action" or behaviors has implications for the construction of a test to measure problem solving ability." (p. 130) This item is included because the usefulness of construing problem solving behavior as a selective information process is indicated, even though the main concern of the paper is the construction of the test.

14. Packer, C. Kyle, and Packer, Toni, "Cybernetics, Information Theory and the Educative Process," Teachers College Record, Vol. 61, 1959, pp. 134-142.

The relevance of feedback, selective information, and coupling for education is discussed. ". . . in a system involving feedback . . . the pupil's performance is taken as part of the information on which the teacher continues to act, and some of this information coming back to the pupil, is the difference between the pupil's actual performance and the given pattern." (p. 137) "The set of possible messages is a different base of operation for the message. Clearly, then, the teacher's function includes knowing not merely the subject, or message, but the other possibilities - the set - from which this message is selected." (p. 139) ". . . the teacher and the student are linked in a system in which . . . reciprocal communication is of utmost importance." (p. 140)

15. Ryans, David G., "An Information-Systems Approach to Education," Santa Monica, Calif.: Systems Development Corporation, TM-1495, 1963.

Education is viewed as a general open system with inputs and outputs, and as having subsystems. The subsystems are subdivisions, e.g. classrooms. All systems exchange information either among elements or subsystems or with other systems. 'Information' is defined in terms of "signs that have semantic and/or pragmatic referents . . ." (p. 6). The system characteristics of steady state, cumulative modifiability, and equifinality are indicated as those of educational information systems. A theory of instruction is devised from the general system-information theory context--one which emphasizes "(a) the interdependence and interrelatedness of conditions and operations influencing teaching-learning; (b) the information processing nature of what goes on when a teacher . . . reaches decisions about programs and instructional behavior (and what similarly goes on when a pupil receives [and assimilates] information from a teacher or instructional instrument . . .; and (c) the information exchange . . . nature of all instruction" (p. 11). The theory is presented as an information flow paradigm, and the explication is with reference to this paradigm. A specific educational system constructed from the theory is described.

16. Ryans, David G., "An Information-System Approach to Theory of Instruction with Special Reference to the Teacher," Santa Monica, Calif.: System Development Corporation, SP-1079, 1963.

General systems theory, selective information theory, a teacher characteristics study, and a theory of behavior whose unit of behavior is dyadic are summarized. The theories and study are taken as the basis for an information-system theory of the instructional process. The definition of selective information is rejected for one taken to be broader (see item 15). The teacher is viewed as a system of three subsystems: external information inputs, internal information inputs and information processing capabilities, and teacher information processing. The components of the first two subsystems are defined topically, e.g. behavior-content, culture, general capabilities, etc. Although two of the topically defined components of the third subsystem are couched in selective information theoretic terms, e.g. input processing: sensing, filtering, etc., the role of selective information is never explicated. The system is presented as an information flow diagram, and this diagram is used to formulate general functional relations between the components. The general system characterizations which are used in the theory are a) the teacher as a system of subsystems of interaction components, b) input, output, and feedback of the system, subsystems, and components, and c) the teacher as an open, self-organizing, self-regulating system. A similar theory is developed for the pupil and the situation complex. The learning process is then represented as an interaction of the teacher system, and the pupil system and the situation complex.

17. Ryans, David G., "The Application of Programmed Instruction and Auto-Instructional Devices in Colleges and their Relation to a Theory of Instruction," Santa Monica, Calif.: System Development Corporation, SP-1084/000/01, 1963.

"This statement is an attempt to look briefly at the purposes, essential characteristics, and potential utility of "programmed instruction" and, especially, to recommend that research and practice with respect to programmed instruction be viewed within a context provided by a theoretical model the author has chosen to label an "information system theory of teaching-learning."" (p. 1) A summary of this theory is presented. For a more complete description of the theory, see items 15 and 16.

18. Ryans, David G., "A Model of Instruction Based on Information Systems Concepts," Theories of Instruction, ed. by James B. Macdonald and Robert R. Leeper, Washington, D. C.: Association for Supervision and Curriculum Development, 1965, pp. 36-61.

This paper is a concise statement of the theory of instruction presented in items 15 and 16. In addition several areas of educational research suggested by the theory are indicated.

19. Thomas, J. Alan, "Administrative Rationality and the Productivity of School Systems," Chicago: Midwest Administration Center, The University of Chicago, 1964, Mimeographed Paper, 52 pp.

It is asserted that "the administrator plays a direct role in affecting the productivity of educational systems" (p. 2). This assertion is analyzed by defining 'system productivity' as a relationship between outputs and inputs. Specific outputs and inputs are cited, and three hypotheses relating these variables are presented: 1) "Short-term (performance-type) outputs are a function of levels of resource inputs; the characteristics of students as indexed by variables describing the home and community from which they come; the knowledge which is brought to bear on the solution of educational problems; and an error term" (p. 28), 2) "Intermediate outcomes (e.g. college-going, success in college, level of schooling completed) are functions of mean test scores; levels of resource input; characteristics of students; knowledge; and an error term" (p. 28), and 3) "Long-term outputs (e.g. Mean income N years after graduation) are functions of intermediate outputs (e.g. years of school completed); short term outputs (mean achievement in school); resource inputs; characteristics of students; knowledge; and an error term" (pp. 28-29).

20. Travers, Robert M. W., "On the Transmission of Information to Human Receivers," Theories of Instruction, ed. by James B. Macdonald and Robert R. Leeper, Washington, D. C.: Association for Supervision and Curriculum Development, 1965, pp. 18-35.

A summary of the research on human information processing is presented. In most of the paper information is taken to be sensory input. The relevance of this body of knowledge to education is indicated by references to educational practices which do not utilize the principles of information processing discussed. However, the framing of classroom situations in terms of informational constructs has significance for the construction of educational theory. For example, the teacher should be aware that in any sensory channel there is noise associated with informational input, and it is possible to code information. A restatement of a postulate of the

> progressive education movement in informational terms is relevant to a selective information context for education: ". . . given several sources of information from which to choose, the pupil would select those which provided information related to the satisfaction of his needs" (p. 30).

State of Usage

Only one use of set theory (item 4) and one of graph theory (item 7) in the construction of educational theory could be discovered. The educational literature dealing with the sociometric analysis of classroom groups is relevant to graph theory, since a sociogram is representable by a graph. In this literature, however, the relevance is not indicated. An example study of this type is Cook's sociometric diagrams in which he exhibits the structural changes within a classroom when attempts are made to modify its structure.[2] Studies similar to Cook's do not provide a basis for the use of graph theory in the construction of educational theory.

Both information and general systems theory have been utilized to a greater extent than set theory and graph theory in the construction of educational theory. Nevertheless, the actual use has not been extensive.

As a general rule, the selective aspect of information theory has been neglected. Consider this quotation from an analysis of classroom interaction:

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[2] L. A. Cook, "An Experimental Socio-graphic Study of a Stratified 10th Grade Class," American Sociological Review, Vol. 10, 1945, pp. 250-261.

> . . . attention will be focused mainly upon the cognitive content of communication as a vehicle for analysis . . . If "information" is used as a general descriptive term denoting the facts, concepts, . . . etc., about which communication takes place, we may make a distinction between sending and receiving [as behavior categories] . . .[3]

Consider also items 2, 15, 16, and 17 of the annotated bibliography.

In regard to the utilization of general systems theory, Ryans' work (items 15 through 18) is an excellent example. Although other examples appear in the bibliography, they are not numerous. This lack is due to exclusion of items on the basis that general systems theory is not utilized. Simply employing the term, 'system', does not necessarily involve such utilization. An example would be theorizing about conceptual systems[4] which has led to theorizing about education[5]. Furthermore, employment of more terms from general systems theory other than 'system' does not necessarily insure utilization. The interrelations possible through the use of such terms do not necessarily accompany a shift to general systems terminology. An illustration among the many in the literature would be a theory of spelling which is called by the authors 'a non-mathematical model'.[6] Input (sources of spelling and linguistics), throughput (channel and register), and output (spelling in action) is put together without involving the interrelations possible through such a shift in terminology. Moreover, this illustration is instructive in that cybernetic terminology is employed. Supposing that cybernetics had been used in a comprehensive manner, nonetheless thereby general systems theory would not be used in a like manner. General systems theory is a more general one which incorporates cybernetics. Cybernetics treats of only one aspect of systems, i.e. the goverance of input by output. Thomas' theorizing cited in item 19 centers about the cybernetic dimension of general systems theory. Finally, systems analysis is not general systems theory, and so literature in education in which the emphasis is on only the selection and arrangement of components within an educational system for efficient realization of a given outcome or outcomes is not cited in the annotated bibliography. T. B. Greenfield, for instance, treats the selected outcome, achievement on departmental examinations, in relation to components within an educational system, pupils within the schools of a school district.[7]

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[3] W. W. Lewis and John M. Newell, "Analysis of Classroom Interaction through Communication Behaviors," Journal of Experimental Education, Vol. 30, 1962, p. 321.

[4] O. J. Harvey, David E. Hunt, and Harold M. Schroder, Conceptual Systems and Personality Organization, New York: John Wiley and Sons, 1961.

[5] Bruce R. Joyce, "A Summary of Exploratory Research in Education Utilizing Conceptual Systems Theory," Chicago: University of Chicago, 1964, Mimeographed Paper, 25 pp.

[6] Paul R. Hanna and Richard E. Hodges, "Spelling and Communications Theory: A Model and an Annotated Bibliograpy," Elementary English, Vol. 40, 1963, pp. 483-505.

Except in the earlier work (item 9) of one author of this report, there are no attempts to use as a basis for constructing educational theory combinations of the theories in the SIGGS Theory Model. One monograph[8], whose author appears to do so, lacks conceptual clarity. A cogent example of the confusion is the equating of general systems theory, cybernetics and information theory. In Ryans' work (items 15 through 18), although his educational theory appears to be devised from a general systems-information theoretic context, selective information theory is not used. The definition of information in item 15 substantiates such non-usage.

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[7] "Administration and System Analysis," The Canadian Administrator, Vol. 3, 1964, pp. 25-30.

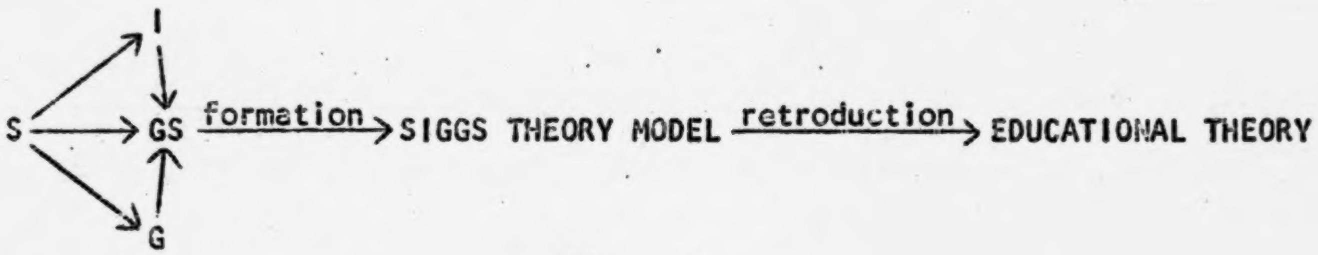
[8] Robert L. Granger, Educational Administration and Information Process, Minneapolis: Department of Educational Administration, College of Education, University of Minnesota, 1965.

The conclusion of this discussion of the state of usage of set theory, information theory, graph theory, and general systems theory in constructing educational theory is twofold:

1. the usage of none of the theories taken singly is in a very advanced stage of development; and
2. there have been no attempts to use a combination of all four theories.

Use in Project

In this project set theory (S), information theory (I), and graph theory (G) are interrelated with general systems theory (GS) to form a theory model (the SIGGS Theory Model). This theory model then is used to retroduce an educational theory. Consequently, this project is not only a first attempt to use a combination of all four theories in the construction of educational theory but is also a construction of educational theory through the theory models approach. See Schema 1 below.



Schema 1

Although the schema summarizes the use in the project and the theory models approach to educational theory construction has been explicated in an earlier project[9], a short resume will be presented in the interest of clarity.

A model for is a characterization used to develop yet another characterization. Because what is being considered is a theory model, a model for theory, it must consist of a group of related comprehensive general characterizations, i.e. it must be theoretical in nature. To illustrate the criteria of relatedness (coherence), comprehensiveness, and generality, psychological theorizing will be contrasted with characterizing Mr. X's behavior. In the former, the attempt is to characterize all aspects of behavior (comprehensiveness) and conditions thereof (relatedness) of any man at any time and any place (generality). In the latter, there may be relatedness but not comprehensiveness (Mr. X's behavior does not exhibit all the behavior possible to a man) or generality (Mr. X's behavior is not the behavior of any man and his behavior occurs at a given time and in a given place).

The theory model is formed from other theories. In this project, portions of set theory as set forth in Chapter I, of information theory in Chapter II, and of graph theory in Chapter III are integrated with general systems theory to form a model, the SIGGS Theory Model set forth in Chapter IV.

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[9] E. S. and G. S. Maccia, "Section I. Metatheory: Retroduction, Models, and Educational Theorizing," Construction of Educational Theory Models, Washington, D. C.: Cooperative Research Project No. 1632, Office of Education, U. S. Department of Health, Education, and Welfare, 1963, pp. 1-100.

From this theory model the educational theory, set forth in the next chapter, is retroduced. To be retroduced means that content is added to the theory model to form the educational theory. It is not the case that educational theory is reduced to the theory of the model, nor that educational theory is deduced from the theory of the model. For example, since a system is a group of components with at least one affect relation which has information, a typology of educational groups which are systems is devised as well as typologies of the kinds of components, affect relations, and information of each kind of educational group.

The educational theory that is retroduced, of course, attempts to meet the criteria of coherence, comprehensiveness, and generality. Moreover, it is important to note that the educational theory is scientific not philosophical in nature. The educational theory consists of hypotheses, amenable to checking through observational data, about education; it does not consist of hypotheses about desirable outcomes of education, which hypotheses are not so amenable. Furthermore, the educational theory is not praxiological[10] in nature, although such a theory could be scientific. In praxiological educational theorizing the purpose is to develop hypotheses about educational practices. Our purpose, however, is non-praxiological although scientific. We are concerned to set forth hypotheses not about how some given outcome or outcomes of education can be attained but rather about human behavior and other factors involved in education irrespective of selected outcomes of education.

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[10] The term, 'praxiological' is taken from Tadeusz Kotarbinski's "Praxiological Sentences and How They are Proved," Logic, Methodology and Philosophy of Science, edited by Nagel, Suppes, and Tarski, Stanford, California: University of Stanford Press, 1962, pp. 211-223.

CHAPTER VII

THE EDUCATIONAL THEORY

Nature of the Theory Development

In developing the educational theory from the SIGGS Theory Model, the first step is to designate what is to be taken as the system. In the language of the educator, it is usually a grouping of schools, such as the New York City Schools, which is designated as 'a system'. Nevertheless, in this educational theory, the school is taken as the system. Such a taking, however, does not preclude a similar development of the educational theory in which a grouping of schools is taken as the system. In fact, it can be seen readily that the SIGGS Theory Model is a source of a school system educational theory as well.[1]

Since a system is a group--at least two components that form a unit--with at least one affect relation with information, these defining characteristics of a system must be given meaning in terms of a school. In other words, kinds of school components, school affect relations, and school information must be specified. Typologies are required.

After a school has been given meaning as a system, then the properties of a system can be given meaning in terms of a school.

Finally, relationships between school properties, i.e. hypotheses about a school, can be set forth. This set of hypotheses, of course, constitutes the educational theory of a school.

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[1] In this paragraph a rudimentary typology of educational groups which are systems is set forth: school "systems" and schools are two such educational groups.

Typologies

Position of persons, things, and symbolic characterizations is in terms of a network of affect relations. Moreover, since components distinctive of any human system and so of a school are persons, things, and symbolic characterizations, attention must be directed first to kinds of affect relations which determine position of such components in a school.

The kinds of affect relations which are involved in such a determination are in terms of specification in regard to affector roles in a school and also affectee roles in a school. Two typologies of school affect relations, therefore, emerge.

An affector in a school may take a role within the instructional process or the inquiry process or the governing process or the facilitating process which accommodates the other processes. Given one of these specified school affector roles, a binary relation and an associated affectee and, thus, a path of communication may be distinguished. In other words, a kind of school affector affect relation may be noted. The typology of school affector affect relations, therefore, is as follows:

1. Instructional
2. Inquiry
3. Governing
4. Facilitating

An affectee in a school may take the role of one who refers his own self-value upon another (one who identifies with another) or one who values the expertise of another or one who values another because of that person's occupancy of a designated position which carries with it legitimacy or one who values another who presumably can reward him or one who values another who presumably can punish him. Given any of these specified affectee roles, a binary relation and an associated affector and, thus, a path for influence may be distinguished. In other words, a kind of school affectee affect relation may be noted. The typology of school affectee affect relations, therefore, is as follows:

1. Referent
2. Expert
3. Legitimate
4. Reward
5. Punishment

To illustrate how these two affect relation typologies interrelate, consider the affector role of instructing and the affectee role of expecting reward. The affector role of instructing determines a relation between teacher and student which establishes a path of communication, while the affectee role of expecting reward determines a relation between student and teacher which establishes a path for influence of the teacher over the student.

On the basis of the typology arising from specified affector roles, four functional units within a school can be distinguished: the instructional and inquiry units which are productive, and the governing and facilitating units which are supportive. Typologies of components within each of these functional units are as follows:

1. Components of an Instructional Unit
    1.1. Persons
    1.1.1. Teachers
    1.1.2. Students
    1.2. Things
    1.2.1. Teaching Devices
    1.2.2. Learning Devices
    1.3. Symbolic Characterizations
    1.3.1. Knowledge about Instruction
    1.3.2. Curriculum
2. Components of an Inquiry Unit
    2.1. Persons
    2.1.1. Researchers
    2.1.2. Developers
    2.2. Things
    2.2.1. Research Devices
    2.2.2. Development Devices
    2.3. Symbolic Characterizations
    2.3.1. Knowledge about Inquiry
    2.3.2. Knowledge
3. Components of a Governing Unit
    3.1. Persons
    3.1.1. Leaders
    3.1.2. Administrators

3.2. Things

3.2.1. Leadership Devices

3.2.2. Administration Devices

3.3. Symbolic Characterizations

3.3.1. Knowledge about Governing

3.3.2. Polices

4. Components of a Facilitating Unit

4.1. Persons

4.1.1. Planning Staff

4.1.2. Servicing Staff

4.2. Things

4.2.1. Planning Devices

4.2.2. Servicing Devices

4.3. Symbolic Characterizations

4.3.1. Knowledge about Facilitating

4.3.2. Directives

Before directing attention to a typology of school information, it should be noted that the typologies or their further development permit taking a part of a school as a system, provided the perspective is shifted from the school as the system to be considered. Any of the functional units could be taken as a system with the corresponding emergence of an educational theory more limited in scope. For example, if the instructional unit is taken as a system, then an instructional educational theory which is less comprehensive than a school educational theory would emerge. Provided the components and affect relations within the curriculum were designated (an example of the further development of the typologies) and the perspective shifted from the instructional unit to the curriculum as the system to be considered, an even more limited educational theory would emerge, i.e. a theory of curriculum. What is being indicated is that the rudimentary typology of educational groups which are systems (referred to on page 119) can be extended as follows:

1. School "System"

1.1. School

1.1.1. Instructional Unit

1.1.1.1. Persons

1.1.1.1.1. Teachers

1.1.1.1.2. Students

1.1.1.2. Things

1.1.1.2.1. Teaching Devices

1.1.1.2.2. Learning Devices

1.1.1.3. Symbolic Characterizations

1.1.1.3.1. Knowledge about Instruction

1.1.1.3.2. Curriculum

1.1.2. Inquiry Unit

.[2]

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1.1.2.3.2. Knowledge

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[2] Where the elipses occur, substitution is to be made from the typologies of components as was done in the case of the instructional unit.

1.1.3. Governing Unit

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1.1.3.2.3. Policies

1.1.4. Facilitating Unit

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1.1.4.2.4. Directives

To complete the giving of meaning to a system which is a school, a typology of school information is required. Setting forth such a typology involves an iteration of all the typologies except, of course, the one designating the kinds of educational groups which could be systems. The iteration follows from the fact that there are as many kinds of information as there are components and affect relations, since the information is on the group or affect relations of a system. For example, there could be student information, because there could be a distribution of students with respect to a given set of categories, such as categories of achievement.

School Properties

Before presenting the properties of a school, it should be noted that the components not in a school which are considered with respect to a school would be a school's surroundings (a school's negasystem). Whatever in a school's surroundings has at least two components with at least one affect relation which has selective information is a school's environmentness; this property is the first presented. In the presentation of school properties, the numbering as well as the symbols of the properties in the SIGGS Theory Model are retained in order to permit cross reference.[3]

18. school environmentness, $E_{\bar{S}}$

    18.1. School environmentness is a school's surroundings of at least two components with at least one affect relation which has selective information.

20. school environmental changeness, $EC_{\bar{S}}$

    20.1. School environmental changeness is a difference in school environmentness.

22. school demand, TP

    22.1. School demand is school environmentness.

23. school resource, IP

    23.1. School resource is a school with selective information.

24. school supply, FP

    24.1. School supply is a school's surroundings environmentness.

25. school depletion, OP

    25.1. School depletion is a school's surroundings with selective information.

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[3] See Tables 1 and 2 on pages 68 and 69 respectively.

26. school storage, SP

26.1. School storage is a school with school resource that is not school supply.

27. school demand transmission, FI

27.1. School demand transmission is a transmission of school demand to a school.

28. school supply transmission, FO

28.1. School supply transmission is a transmission of school supply to a school's surroundings.

29. school demand transfer, FT

29.1. School demand transfer is a transmission of school demand through a school to its surroundings.

30. school supply transfer, FB

30.1. School supply transfer is a transmission of school supply through a school's surroundings to a school.

31. school filtrationness, FL

31.1. School filtrationness is a restriction of school demand.

32. school spillageness, SL

32.1. School spillageness is a restriction of school demand transmission.

33. school regulationness, RG

33.1. School regulationness is adjustment of school supply.

34. school compatibleness, CP

34.1. School compatibleness is a commonality between school demand transmission and school supply transmission.

35. school openness, O

35.1. School openness is school demand transmission and/or school supply transmission.

36. school adaptiveness, AD

36.1. School adaptiveness is a difference in school compatibleness under school environmental changeness.

37. school efficientness, EF

37.1. School efficientness is commonality between school demand transfer and school demand.

38. school complete connectionness, CC

38.1. School complete connectionness is every two school components directly channeled to each other with respect to school affect relations.

39. school strongness, SR

39.1. School strongness is not school complete connectionness and every two school components are channeled to each other with respect to school affect relations.

40. school unilateralness, U

40.1. School unilateralness is not either school complete connectionness or school strongness and every two school components have a channel between them with respect to school affect relations.

41. school weakness, WE

41.1. School weakness is not either school complete connectionness or school strongness or school unilateralness and every two school components are connected with respect to school affect relations.

42. school disconnectionness, DC

42.1. School disconnectionness is not either school complete connectionness or school strongness or school unilateralness or school weakness and some school components are not connected with respect to affect relations.

43. school vulnerableness, VN

43.1. School vulnerableness is some connections which when removed produce disconnectionness with respect to school affect relations.

44. school passive dependentness, D_P

44.1. School passive dependentness is school components which have channels to them.

45. school active dependentness, D_A

45.1. School active dependentness is school components which have channels from them.

46. school independentness, I

46.1. School independentness is school components which do not have channels to them.

47. school segregationness, SG

47.1. School segregationness is school independentness under school environmental changeness.

48. school interdependentness, ID

48.1. School interdependentness is school components which have channels to and from them.

49. school wholeness, W

49.1. School wholeness is school components which have channels to all other school components.

50. school integrationness, IG

50.1. School integrationness is school wholeness under school environmental changeness.

51. school hierarchically orderness, HO

51.1. School hierarchically orderness is levels of subordinateness with school components in each level with respect to school affect relations.

52. school flexibleness, F

52.1. School flexibleness is different subgroups of school components through which there is a channel between two school components with respect to school affect relations.

53. school homomorphismness, HM

53.1. School homomorphismness is school components having the same connections as other school components.

54. school isomorphismness, IM

54.1. School isomorphismness is school components having the same connections as other corresponding school components.

55. school automorphismness, AM

55.1. School automorphismness is school components whose connections can be transformed so that the same connections hold.

56. school compactness, CO

56.1. School compactness is the average number of direct channels in a channel between school components.

57. school centralness, CE

57.1. School centralness is concentration of channels.

58. school sizeness, SZ

58.1. School sizeness is the number of school components.

59. school complexness, CX

59.1. School complexness is the number of connections.

60. school selective informationness, SI

60.1. School selective informationness is the amount of school selective information.

61. school size growthness, ZG

61.1. School size growthness is increase in school sizeness.

62. school complexity growthness, XG

62.1. School complexity growthness is increase in school complexness.

63. school selective information growthness, TG

63.1. School selective information growthness is increase in school selective informationness.

64. school size degenerationness, ZD

64.1. School size degenerationness is decrease in school sizeness.

65. school complexity degenerationness, XD

65.1. School complexity degenerationness is decrease in school complexness.

66. school selective information degenerationness, TD

66.1. School selective information degenerationness is decrease in school selective informationness.

67. school stableness, SB

67.1. School stableness is no change with respect to school conditions.

68. school state steadiness, SS

68.1. School state steadiness is school stableness under school environmental changeness.

69. school state determinationness, SD

69.1. School state determinationness is derivability of school conditions from one and only one school state.

70. school equifinalness, EL

70.1. School equifinalness is derivability of school conditions from other school states.

71. school homeostasisness, HS

71.1. School homeostasisness is school equifinalness under school environmental changeness.

72. school stressness, SE

72.1. School stressness is change beyond certain limits of school's surroundings state.

73. school strainness, SA

73.1. School strainness is change beyond certain limits of school state.

Relating the typologies to the school properties, it should be obvious that the properties could hold with respect to one or more kinds of affector affect relations and thus with respect to one or more kinds of components, as well as with respect to one or more kinds of affectee affect relations. For instance, a school might have wholeness with respect to instructional components or inquiry components or governing components or facilitating components or any combination thereof, as well as with respect to referent affect relations or expert affect relations or legitimate affect relations or reward affect relations or punishment affect relations or any combination thereof.

Nature of the Hypotheses

The hypotheses are proposed relationships between school properties. 'To be proposed' means that the relationships are in need of verification; they could be false as well as true. Furthermore, nearly all the relationships are dynamic rather than static; that is, a change in one set of properties is specified to entail a change in another set of properties rather than no variation with respect to the properties being involved.

Illustrations should clarify dynamic hypotheses as opposed to static ones. One could propose the hypothesis, HO ⇒ CE, i.e. school hierarchically orderness implies school centralness. Such a proposed relationship is static in that nothing is asserted as to the way in which school centralness varies with school hierarchically orderness. The hypothesis, $\overrightarrow{HO} \wedge W\uparrow \Rightarrow EF\downarrow$, would be a dynamic one. In this hypothesis, not only is it proposed that school hierarchically orderness and school wholeness implies school efficientness but also that school efficientness decreases as school wholeness increases and school hierarchically orderness is constant.

Symbols, such as →, ↑, and ↓, were not utilized in the SIGGS Theory Model. Table 1 presents a list of additional symbols to be used in the hypotheses constituting the educational theory.

Logico-mathematical Symbols	Verbal Symbols
1. $\ldots\uparrow$	... increases
2. $\ldots\downarrow$	... decreases
3. $\ldots\curvearrowright$	... increases to some value and then decreases
4. $\ldots\curvearrowleft$	... decreases to some value and then increases
5. $\underline{\ldots}$	... is greater than some value
6. $\overline{\ldots}$	... is less than some value
7. $\overrightarrow{\ldots}$	... is constant
8. $\Delta\ldots$	change in ...
9. max ...	... is maximum
10. $\underline{\underline{\ldots}}$	... is nearly maximum
11. $\overline{\overline{\ldots}}$	... is nearly minimum

Table 1

Presentation of the Hypotheses

At least two ways of presenting the hypotheses are possible: paralleling the sequencing in the SIGGS Theory Model and according to the interrelations of set theory, information theory and graph theory. In Appendix III, a listing of hypotheses according to the former mode is presented to permit utilization of computers in testing consistency and deducing further hypotheses. It can be noted that the sequence of the SIGGS Theory Model is maintained in the antecedent as well as the conjuncts within the antecedent and within the consequent. In Appendix IV, a listing of hypotheses according to the latter mode is presented to suggest other possible integrations and permit checking by means of data. Both of these listings are in symbolic form. In this chapter, the hypotheses are also presented in the form of propositions in English. In this presentation, the latter mode is utilized and is clarified through the following typology:

1. Information Theoretic Hypotheses

1.1. Information Theoretic Antecedent--Information Theoretic Consequent

2. Graph Theoretic Hypotheses

2.1. Graph Theoretic Antecedent--Graph Theoretic Consequent

2.1.1. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Graph Theoretic Consequent

2.1.2. Both Graph Theoretic Antecedent and Consequent with Respect to Affect Relations

2.2. Graph Theoretic Hypotheses with Respect to Affect Relations

3. Set Theoretic Hypotheses

4. Information and Graph Theoretic Hypotheses

4.1. Information Theoretic Antecedent--Graph Theoretic Consequent

4.1.1. Information Theoretic Antecedent--Graph Theoretic Consequent with Respect to Affect Relation

4.2. Graph Theoretic Antecedent--Information Theoretic Consequent

4.2.1. Graph Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

4.2.2. Both Graph Theoretic Antecedent and Information Theoretic Consequent with Respect to Affect Relations

4.2.3. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information Theoretic Consequent

4.3. Graph Theoretic Antecedent--Information and Graph Theoretic Consequent

4.3.1. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information and Graph Theoretic Consequent

4.4. Information and Graph Theoretic Antecedent--Graph Theoretic Consequent

4.5. Information and Graph Theoretic Antecedent--Information Theoretic Consequent

4.5.1. Information and Graph Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

5. Information and Set Theoretic Hypotheses

5.1. Information Theoretic Antecedent--Set Theoretic Consequent

5.1.1. Both Information Theoretic Antecedent and Set Theoretic Consequent with Respect to Affect Relations

5.2. Set Theoretic Antecedent--Information Theoretic Consequent

5.2.1. Set Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

5.3. Information and Set Theoretic Antecedent--Information Theoretic Consequent

5.4. Information and Set Theoretic Antecedent--Set Theoretic Consequent

6. Graph and Set Theoretic Hypotheses

6.1. Graph Theoretic Antecedent--Set Theoretic Consequent

6.1.1. Both Graph Theoretic Antecedent and Set Theoretic Consequent with Respect to Affect Relation

6.2. Set Theoretic Antecedent--Graph Theoretic Consequent

6.3. Graph Theoretic Antecedent--Graph and Set Theoretic Consequent

6.4. Set Theoretic Antecedent--Graph and Set Theoretic Consequent

6.5. Graph and Set Theoretic Antecedent--Graph Theoretic Consequent

6.6. Graph and Set Theoretic Antecedent--Set Theoretic Consequent

7. Information, Graph, and Set Theoretic Hopotheses

7.1. Graph Theoretic Antecedent--Information and Set Theoretic Consequent

7.1.1. Graph Theoretic Antecedent with Respect to Affect Relations--Information and Set Theoretic Consequent with Respect to Affect Relation

7.2. Set Theoretic Antecedent--Information, Graph, and Set Theoretic Consequent

7.3. Information and Graph Theoretic Antecedent--Set Theoretic Consequent

7.4. Information and Set Theoretic Antecedent--Graph Theoretic Consequent

7.5. Graph and Set Theoretic Antecedent--Information Theoretic Consequent

School Hypotheses

1. Information Theoretic Hypotheses

1.1. Information Theoretic Antecedent--Information Theoretic Consequent

1a. If school environmental changeness increases, then change in school resource is greater than some value.

1b. $EC_{\bar{S}}\uparrow \Rightarrow \underline{\Delta IP}$

2a. If school environmental changeness increases, then change in school supply is greater than some value.

2b. $EC_{\bar{S}}\uparrow \Rightarrow \underline{\Delta FP}$

3a. If school environmental changeness increases, then change in school supply transfer is greater than some value.

3b. $EC_{\bar{S}}\uparrow \Rightarrow \underline{\Delta FB}$

4a. If school environmental changeness increases, then change in school filtrationness is greater than some value.

4b. $EC_{\bar{S}}\uparrow \Rightarrow \underline{\Delta FL}$

5a. If school demand increases, then school resource increases to some value and then decreases.

5b. $TP\uparrow \Rightarrow IP\cap\!\downarrow$

6a. If school demand greater than some value increases, then school supply increases.

6b. $\underline{TP}\uparrow \Rightarrow FP\uparrow$

7a. If school demand is nearly minimum, then school supply increases.

7b. $\overline{\overline{TP}} \Rightarrow FP\uparrow$

8a. If school demand increases, then school filtrationness decreases to some value and then increases.

8b. $TP\uparrow \Rightarrow FL\cup\!\uparrow$

9a. If school demand increases, then school regulationness less than some value increases.

9b. $TP\uparrow \Rightarrow \overline{RG}\uparrow$

10a. If school resource decreases, then school supply decreases.

10b. IP↓ ⇒ FP↓

11a. If school resource decreases, then school storage decreases.

11b. IP↓ ⇒ SP↓

12a. If school resource increases, then school filtrationness decreases.

12b. IP↑ ⇒ FL↓

13a. If school resource decreases, then school filtrationness increases.

13b. IP↓ ⇒ FL↑

14a. If school resource is greater than some value, then school regulationness is greater than some value.

14b. $\underline{IP} \Rightarrow \underline{RG}$

15a. If school depletion increases, then school supply increases.

15b. OP↑ ⇒ FP↑

16a. If school storage decreases, then school supply transmission decreases.

16b. SP↓ ⇒ FO↓

17a. If school storage increases, then school adaptiveness increases.

17b. SP↑ ⇒ AD↑

18a. If school storage increases, then school efficientness decreases.

18b. SP↑ ⇒ EF↓

19a. If school demand transmission increases, then school supply increases to some value and then decreases.

19b. $FI\uparrow \Rightarrow FP\curvearrowright$

20a. If school demand transmission increases, then school spillageness increases.

20b. $FI\uparrow \Rightarrow SL\uparrow$

21a. If school demand transfer increases, then school compatibleness increases.

21b. $FT\uparrow \Rightarrow CP\uparrow$

22a. If school demand transfer is less than some value, then school filtrationness is greater than some value or school spillageness is greater than some value.

22b. $\overline{FT} \Rightarrow \underline{FL} \vee \underline{SL}$

23a. If change in school supply transfer is greater than some value, then school environmental changeness increases.

23b. $\underline{\Delta FB} \Rightarrow EC_{\overline{S}}\uparrow$

24a. If school supply transfer is greater than some value, then school storage is less than some value.

24b. $\underline{FB} \Rightarrow \overline{SP}$

25a. If school supply transfer is greater than some value, then school regulationness is less than some value.

25b. $\underline{FB} \Rightarrow \overline{RG}$

26a. If school filtrationness is greater than some value, then school compatibleness is greater than some value.

26b. $\underline{FL} \Rightarrow \underline{CP}$

27a. If school filtrationness is less than some value, then school compatibleness is less than some value.

27b. $\overline{FL} \Rightarrow \overline{CP}$

28a. If school filtrationness increases, then school adaptiveness increases.

28b. $FL\uparrow \Rightarrow AD\uparrow$

29a. If school openness increases, then school efficientness decreases.

29b. $O\uparrow \Rightarrow EF\downarrow$

30a. If school environmental changeness increases and school supply increases, then change in school supply transmission is greater than some value.

30b. $EC_{\overline{S}}\uparrow \wedge FP\uparrow \Rightarrow \underline{\Delta FO}$

31a. If school environmental changeness increases and school supply increases, then change in school demand transfer is greater than some value.

31b. $EC_{\overline{S}}\uparrow \wedge FP\uparrow \Rightarrow \underline{\Delta FT}$

32a. If school environmental changeness and school demand transfer is greater than some value, then school stableness is greater than some value.

32b. $\underline{EC_{\overline{S}}} \wedge \underline{FT} \Rightarrow \underline{SB}$

33a. If school demand increases and school supply increases, then school demand transfer increases.

33b. $TP\uparrow \wedge FP\uparrow \Rightarrow FT\uparrow$

34a. If school demand is constant and school efficientness is greater than some value, then school regulationness is less than some value.

34b. $T\vec{P} \wedge \underline{EF} \Rightarrow \overline{RG}$

35a. If school resource is constant and school supply is constant, then school depletion is constant.

35b. $I\vec{P} \wedge F\vec{P} \Rightarrow O\vec{P}$

36a. If school resource increases and school storage is constant, then school supply transmission increases.

36b. $IP\uparrow \wedge S\vec{P} \Rightarrow FO\uparrow$

37a. If school resource increases and school storage is less than some value, then change in school resource is equal to change in school storage.

37b. $IP\uparrow \wedge \overline{SP} \Rightarrow \Delta IP = \Delta SP$

38a. If change in school resource is greater than change in school demand transfer, then school spillageness increases.

38b. $\Delta IP > \Delta FT \Rightarrow SL\uparrow$

39a. If school resource is greater than some value and school spillageness is less than some value, then school storage increases.

39b. $\underline{IP} \wedge \overline{SL} \Rightarrow SP\uparrow$

40a. If school resource is less than some value and school spillageness is less than some value, then school storage decreases.

40b. $\overline{IP} \wedge \overline{SL} \Rightarrow SP\downarrow$

41a. If school resource is constant and school efficientness at a given time is less than some value, then school efficientness increases.

41b. $I\vec{P} \wedge \overline{EF(t_1)} \Rightarrow EF\uparrow$

42a. If the ratio of maximum school selective informationness to school resource decreases, then school supply transmission decreases.

42b. $\frac{\max SI}{IP}\downarrow \Rightarrow FO\downarrow$

43a. If school supply increases and school depletion is less than some value, then school supply transmission decreases.

43b. $FP\uparrow \wedge \overline{OP} \Rightarrow FO\downarrow$

44a. If change in school supply is less than some value and change in school storage is less than zero and change in school supply is greater than zero and the negative of change in school storage is greater than some value, then school efficientness decreases.

44b. $\overline{\Delta FP} \wedge \Delta SP < 0 < \Delta FP \wedge -\underline{\Delta SP} \Rightarrow EF\downarrow$

45a. If school depletion increases and school supply transfer is greater than some value, then school resource increases.

45b. $OP\uparrow \wedge \underline{FB} \Rightarrow IP\uparrow$

46a. If school storage increases and school filtrationness decreases or school spillageness decreases, then school information growthness increases.

46b. $SP\uparrow \wedge .\ FL\downarrow \vee SL\downarrow\ . \Rightarrow TG\uparrow$

47a. If school demand transfer is greater than some value and school spillageness is less than some value and school supply transfer is greater than some value, then school efficientness is greater than some value.

47b. $\underline{FT} \wedge \overline{SL} \wedge \underline{FB} \Rightarrow \underline{EF}$

48a. If school demand transmission increases and school supply transmission is constant and school compatibleness is constant or school demand transmission is constant and school supply transmission increases and school compatibleness is constant or school demand transmission is constant and school supply transmission is constant and school compatibleness decreases, then school openness increases.

48b. $FI\uparrow \wedge F\vec{O} \wedge C\vec{P}\ .\vee.\ F\vec{I} \wedge FO\uparrow \wedge C\vec{P}\ .\vee.\ F\vec{I} \wedge F\vec{O} \wedge CP\downarrow\ .\Rightarrow O\uparrow$

49a. If school demand transmission decreases and school supply transmission is constant and school compatibleness is constant or school demand transmission is constant and school supply transmission decreases and school compatibleness is constant or school demand transmission is constant and school supply transmission is constant and school compatibleness increases, then school openness decreases.

49b. $FI\downarrow \wedge F\vec{O} \wedge C\vec{P}\ .\vee.\ F\vec{I} \wedge FO\downarrow \wedge C\vec{P}\ .\vee.\ F\vec{I} \wedge F\vec{O} \wedge CP\uparrow\ .\Rightarrow O\downarrow$

50a. Change in school resource is greater than change in school supply.

50b. $\Delta IP > \Delta FP$

51a. Change in school demand transmission is greater than change in school supply transmission.

51b. $\Delta FI > \Delta FO$

52a. School efficientness is equal to the maximum school efficientness, if and only if school demand transmission is equivalent to school supply transmission.

52b. $EF = \max EF \Leftrightarrow FI \equiv FO$

2. Graph Theoretic Hypotheses

2.1. Graph Theoretic Antecedent--Graph Theoretic Consequent

53a. If school complete connectionness increases, then school flexibleness increases.

53b. $CC\uparrow \Rightarrow F\uparrow$

54a. If school strongness decreases, then school wholeness increases.

54b. $SR\downarrow \Rightarrow W\uparrow$

55a. If school strongness increases, then school hierarchically orderness decreases.

55b. $SR\uparrow \Rightarrow HO\downarrow$

56a. If school strongness increases, then school flexibleness increases.

56b. $SR\uparrow \Rightarrow F\uparrow$

57a. If school unilateralness, then school hierarchically orderness.

57b. $U \Rightarrow HO$

58a. If school disconnectionness is greater than some value, then school independentness increases.

58b. $\underline{DC} \Rightarrow I\uparrow$

59a. If school disconnectionness is greater than some value, then school segregationness increases.

59b. $\underline{DC} \Rightarrow SG\uparrow$

60a. If school vulnerableness increases, then school complete connectionness decreases.

60b. $VN\uparrow \Rightarrow CC\downarrow$

61a. If school passive dependentness increases, then school centralness increases.

61b. $D_P\uparrow \Rightarrow CE\uparrow$

62a. If school active dependentness increases, then school centralness decreases.

62b. $D_A\uparrow \Rightarrow CE\downarrow$

63a. If school interdependentness increases, then school complexity growthness increases.

63b. $ID\uparrow \Rightarrow XG\uparrow$

64a. If school hierarchically orderness increases, then school vulnerableness increases and school flexibleness decreases.

64b. $HO\uparrow \Rightarrow VN\uparrow \wedge F\downarrow$

65a. If school compactness increases, then school hierarchically orderness decreases.

65b. $CO\uparrow \Rightarrow HO\downarrow$

66a. If school centralness increases, then school passive dependentness increases.

66b. $CE\uparrow \Rightarrow D_P\uparrow$

67a. If school centralness increases, then school active dependentness decreases.

67b. $CE\uparrow \Rightarrow D_A\downarrow$

68a. If school centralness is less than some value, then school independentness increases.

68b. $\overline{CE} \Rightarrow I\uparrow$

69a. If school centralness is less than some value, then school centralness increases.

69b. $\overline{CE} \Rightarrow CE\uparrow$

70a. If school wholeness increases and school hierarchically orderness is constant, then school integrationness increases.

70b. $W\uparrow \wedge \overrightarrow{HO} \Rightarrow IG\uparrow$

71a. The limit of the ratio of school active dependentness to school passive dependentness as school unilateralness increases is equal to one.

71b. $\lim\limits_{U\uparrow} \frac{D_A}{D_P} = 1$

2.1.1. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Graph Theoretic Consequent

72a. If maximum school passive dependentness with respect to governing and legitimate affect relations, then school wholeness increases and school hierarchically orderness increases and school centralness increases.

72b. $\max D_{P\ \text{Gov.-Leg.}} \Rightarrow W\uparrow \wedge HO\uparrow \wedge CE\uparrow$

2.1.2. Both Graph Theoretic Antecedent and Consequent with Respect to Affect Relations

73a. If school strongness with respect to governing affect relation, then school complete connectionness with respect to school referent affect relation.

73b. $SR_{\text{Gov.}} \Rightarrow CC_{\text{Ref.}}$

74a. If school strongness with respect to referent affect relation, then school vulnerableness with respect to governing affect relation decreases.

74b. $SR_{Ref.} \Rightarrow VN_{Gov.}\downarrow$

75a. If school strongness with respect to referent affect relation, then school vulnerableness with respect to referent affect relation decreases.

75b. $SR_{Ref.} \Rightarrow VN_{Ref.}\downarrow$

76a. If school strongness with respect to reward affect relation is greater than some value, then school complete connectionness with respect to referent affect relation increases or school strongness with respect to referent affect relation increases.

76b. $\underline{SR}_{Rew.} \Rightarrow CC_{Ref.}\uparrow \vee SR_{Ref.}\uparrow$

77a. If school strongness with respect to reward affect relation is greater than some value, then school wholeness with respect to governing affect relation and school hierarchically orderness with respect to governing affect relation.

77b. $\underline{SR}_{Rew.} \Rightarrow W_{Gov.} \wedge HO_{Gov.}$

78a. If school strongness with respect to governing affect relation increases and school hierarchically orderness with respect to governing affect relation decreases, then school strongness with respect to referent affect relation increases.

78b. $SR_{Gov.}\uparrow \wedge HO_{Gov.}\downarrow \Rightarrow SR_{Ref.}\uparrow$

79a. If school strongness with respect to referent affect relation is greater than some value, and school hierarchically orderness with respect to governing affect relation is greater than some value, then school wholeness with respect to governing affect relation.

79b. $\underline{SR}_{Ref.} \wedge \underline{HO}_{Gov.} \Rightarrow W_{Gov.}$

80a. If school strongness with respect to referent affect relation is less than some value and school centralness with respect to governing affect relation, then school wholeness with respect to governing affect relation.

80b. $\overline{SR}_{Ref.} \wedge CE_{Gov.} \Rightarrow W_{Gov.}$

81a. If school strongness with respect to referent affect relation is less than some value, and school hierarchically orderness with respect to governing affect relation is greater than some value and school centralness with respect to governing affect relation, then school compactness with respect to governing affect relation increases.

81b. $\overline{SR}_{Ref.} \wedge \underline{HO}_{Gov.} \wedge CE_{Gov.} \Rightarrow CO_{Gov.}\uparrow$

82a. If school wholeness with respect to referent affect relation, then school complete connectionness with respect to referent affect relation increases or school strongness with respect to referent affect relation increases.

82b. $W_{Ref.} \Rightarrow CC_{Ref.}\uparrow \vee SR_{Ref.}\uparrow$

83a. If school hierarchically orderness with respect to governing affect relation is greater than some value and school flexibleness with respect to governing affect relation is greater than some value, then school disconnectionness with respect to referent affect relation.

83b. $\underline{HO}_{Gov.} \wedge \underline{F}_{Gov.} \Rightarrow DC_{Ref.}$

2.2. Graph Theoretic Hypotheses with Respect to Affect Relations

84a. School disconnectionness is greater than some value with respect to instructional affect relation.

84b. $\underline{DC}_{Ins.}$

85a. School disconnectionness is greater than some value with respect to inquiry affect relation.

85b. $\underline{DC}_{Inq.}$

3. Set Theoretic Hypotheses

86a. If school state steadiness is greater than some value, then school strainness increases.

86b. $\underline{SS} \Rightarrow SA\uparrow$

87a. If school stressness is less than some value, then school state steadiness is constant.

87b. $\overline{SE} \Rightarrow S\vec{S}$

88a. If school stressness greater than some value increases, then school strainness increases.

88b. $\underline{SE}\uparrow \Rightarrow SA\uparrow$

89a. School state steadiness increases if and only if school state determinationness increases, and school state steadiness decreases if and only if school state determinationness decreases.

89b. $SS\uparrow \Leftrightarrow SD\uparrow \ .\wedge.\ SS\downarrow \Leftrightarrow SD\downarrow$

4. Information and Graph Theoretic Hypotheses

4.1. Information Theoretic Antecedent--Graph Theoretic Consequent

90a. If school demand increases, then school centralness decreases.

90b. $TP\uparrow \Rightarrow CE\downarrow$

91a. If school demand transmission decreases, then school unilateralness decreases.

91b. $FI\downarrow \Rightarrow U\downarrow$

92a. If school demand transmission less than some value decreases, then school hierarchically orderness decreases.

92b. $\overline{FI}\downarrow \Rightarrow HO\downarrow$

93a. If school demand transmission decreases, then school complexity degenerationness increases.

93b. $FI\downarrow \Rightarrow XD\uparrow$

94a. If school supply transmission is less than some value, then school complexity degenerationness increases.

94b. $\overline{FO} \Rightarrow XD\uparrow$

95a. If school demand transfer increases, then school weakness is less than some value.

95b. $FT\uparrow \Rightarrow \overline{WE}$

96a. If school demand is nearly minimum and school supply increases, then school disconnectionness increases.

96b. $\overline{\overline{TP}} \wedge FP\uparrow \Rightarrow DC\uparrow$

97a. If school demand transmission increases and school compatibleness is nearly minimum, then school disconnectionness increases.

97b. $FI\uparrow \wedge \overline{\overline{CP}} \Rightarrow DC\uparrow$

98a. If school storage increases and school filtrationness decreases or school spillageness decreases, then school integrationness increases.

98b. $SP\uparrow \wedge .\ FL\downarrow \vee SL\downarrow\ . \Rightarrow IG\uparrow$

4.1.1. Information Theoretic Antecedent--Graph Theoretic Consequent with Respect to Affect Relation

99a. If school resource increases and school storage is greater than some value, then school segregationness with respect to referent affect relation.

99b. $IP\uparrow \wedge \underline{SP} \Rightarrow SG_{Ref.}$

4.2. Graph Theoretic Antecedent--Information Theoretic Consequent

100a. If school complete connectionness increases, then school demand transmission increases.

100b. $CC\uparrow \Rightarrow FI\uparrow$

101a. If school weakness is greater than some value, then school demand transfer is less than some value.

101b. $\underline{WE} \Rightarrow \overline{FT}$

102a. If school interdependentness increases, then school demand transmission increases.

102b. $ID\uparrow \Rightarrow FI\uparrow$

103a. If school wholeness increases, then school regulationness is less than some value.

103b. $W\uparrow \Rightarrow \overline{RG}$

104a. If school compactness greater than some value increases, then school efficientness increases.

104b. $\underline{CO}\uparrow \Rightarrow EF\uparrow$

105a. If school centralness increases, then school demand decreases.

105b. $CE\uparrow \Rightarrow TP\downarrow$

106a. If school complete connectionness increases or school strongness increases, then school demand increases.

106b. $CC\uparrow \lor SR\uparrow \Rightarrow TP\uparrow$

107a. If school complete connectionness increases or school strongness increases, then school resource increases.

107b. $CC\uparrow \lor SR\uparrow \Rightarrow IP\uparrow$

108a. If school complete connectionness increases or school strongness increases, then school filtrationness decreases.

108b. $CC\uparrow \lor SR\uparrow \Rightarrow FL\downarrow$

109a. If school complete connectionness increases or school strongness increases, then school spillageness increases.

109b. $CC\uparrow \lor SR\uparrow \Rightarrow SL\uparrow$

110a. If school complete connectionness increases or school strongness increases, then zero is less than change in school supply and change in school supply is less than change in school resource.

110b. $CC\uparrow \lor SR\uparrow \Rightarrow 0 < \Delta FP < \Delta IP$

111a. If school completc connectionness increases or school strongness increases, then change in school storage is greater than change in school supply.

111b. $CC\uparrow \vee SR\uparrow \Rightarrow \Delta SP > \Delta FP$

112a. If school strongness increases and school hierarchically orderness is constant, then school regulationness decreases.

112b. $SR\uparrow \wedge \vec{HO} \Rightarrow RG\downarrow$

113a. If school wholeness increases and school hierarchically orderness is constant, then school efficientness decreases.

113b. $W\uparrow \wedge \vec{HO} \Rightarrow EF\downarrow$

114a. If school weakness and school hierarchically orderness, then school flexibleness decreases.

114b. $WE \wedge HO \Rightarrow F\downarrow$

115a. If school unilateralness or school weakness increases or school disconnectionness increases, then school resource decreases and school supply decreases.

115b. $U \vee WE\uparrow \vee DC\uparrow \Rightarrow IP\downarrow \wedge FP\downarrow$

4.2.1. Graph Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

116a. If school passive dependentness with respect to reward affect relation increases, then school supply transmission decreases.

116b. $D_{P\ Rew.}\uparrow \Rightarrow FO\downarrow$

117a. If school passive dependentness with respect to reward affect relation increases, then school adaptiveness greater than some value increases.

117b. $D_{P\ Rew.}\uparrow \Rightarrow \underline{AD}\uparrow$

118a. If school independentness with respect to governing affect relation increases, then school supply increases.

118b. $I_{Gov.}\uparrow \Rightarrow FP\uparrow$

119a. If school independentness with respect to governing affect relation increases, then school depletion is less than some value.

119b. $I_{Gov.}\uparrow \Rightarrow \overline{OP}$

120a. If school independentness with respect to governing affect relation increases, then school supply transmission decreases.

120b. $I_{Gov.}\uparrow \Rightarrow FO\downarrow$

121a. If school wholeness with respect to referent affect relation is greater than some value, then the absolute value of the difference of school supply from maximum school supply is greater than some value.

121b. $\underline{W}_{Ref.} \Rightarrow \underline{|\max FP - FP|}$

122a. If school wholeness with respect to referent affect relation is greater than some value, then school openness is nearly minimum.

122b. $\underline{W}_{Ref.} \Rightarrow \overset{=}{\overline{O}}$

123a. If school hierarchically orderness with respect to governing affect relation increases, then school filtrationness increases.

123b. $HO_{Gov.}\uparrow \Rightarrow FL\uparrow$

124a. If school complexness with respect to facilitating affect relation is greater than some value, then school regulationness is greater than some value.

124b. $\underline{CX}_{Fac.} \Rightarrow \underline{RG}$

4.2.2. Both Graph Theoretic Antecedent and Information Theoretic Consequent with Respect to Affect Relation

125a. If school complexness with respect to facilitating affect relation is greater than some value, then school demand transfer with respect to facilitating affect relation is less than some value.

125b. $\underline{CX}_{Fac.} \Rightarrow \overline{FT}_{Fac.}$

4.2.3. Graph Theoretic Antecedent with Respect to Two Affect Relations--Information Theoretic Consequent

126a. If school passive dependentness with respect to inquiry and legitimate affect relations increases, then school supply transmission increases and school spillageness increases and maximum school selective informationness is greater than some value.

126b. $D_{P\ Inq.-Leg.}\uparrow \Rightarrow FO\uparrow \wedge SL\uparrow \wedge \underline{\max SI}$

127a. If school passive dependentness with respect to inquiry and expert affect relations increases, then school supply transmission decreases and school spillageness greater than some value increases and maximum school selective informationness is less than some value.

127b. $D_{P\ Inq.-Exp.}\uparrow \Rightarrow FO\downarrow \wedge \underline{SL}\uparrow \wedge \overline{\max SI}$

128a. If school active dependentness with respect to facilitating and legitimate affect relations is greater than some value, then school regulationness is less than some value.

128b. $\underline{D_A}\ Fac.-Leg. \Rightarrow \overline{RG}$

129a. If school wholeness with respect to inquiry and referent affect relations increases, then the ratio of maximum school selective informationness to school resource increases.

129b. $W_{Inq.-Ref.}\uparrow \Rightarrow \frac{\max SI}{IP}$

130a. If school disconnectionness with respect to instructional and referent affect relations is greater than some value and school complete connectionness with respect to instructional and referent affect relations increases and school wholeness with respect to instructional and referent affect relations increases, then school resource increases and school supply increases and school supply transmission decreases and school regulationness increases.

130b. $\underline{DC}_{\text{Ins.-Ref.}} \wedge CC_{\text{Ins.-Ref.}}\uparrow \wedge W_{\text{Ins.-Ref.}}\uparrow \Rightarrow$

$IP\uparrow \wedge FP\uparrow \wedge FO\downarrow \wedge RG\uparrow$

131a. If school disconnectionness with respect to instructional and expert affect relations is greater than some value and school complete connectionness with respect to instructional and expert affect relations increases and school wholeness with respect to instructional and expert affect relations increases, then school resource increases and school storage increases and school supply transmission increases and school filtrationness increases.

131b. $\underline{DC}_{\text{Ins.-Exp.}} \wedge CC_{\text{Ins.-Exp.}}\uparrow \wedge W_{\text{Ins.-Exp.}}\uparrow \Rightarrow$

$IP\uparrow \wedge SP\uparrow \wedge FO\uparrow \wedge FL\uparrow$

132a. If school disconnectionness with respect to instructional and referent affect relations is greater than some value and school passive dependentness with respect to instructional and referent affect relations increases and school wholeness with respect to instructional and referent affect relations increases, then school resource decreases and school supply decreases and school supply transmission decreases and school regulationness decreases.

132b. $\underline{DC}_{\text{Ins.-Ref.}} \wedge D_{P\ \text{Ins.-Ref.}}\uparrow \wedge W_{\text{Ins.-Ref.}}\uparrow \Rightarrow$

$IP\downarrow \wedge FP\downarrow \wedge FO\downarrow \wedge RG\downarrow$

133a. If school disconnectionness with respect to instructional and reward affect relations is greater than some value and school passive dependentness with respect to instructional and reward affect relations increases and school wholeness with respect to instructional and reward affect relations increases, then if school environmental changeness is greater than some value then school adaptiveness is greater than some value, and school resource is less than some value and school storage is less than some value and school filtrationness is greater than some value.

133b. $\underline{DC}_{\text{Ins.-Rew.}} \wedge D_P\ \text{Ins.-Rew.}\uparrow \wedge W_{\text{Ins.-Rew.}}\uparrow \Rightarrow. \underline{EC}_{\overline{S}} \Rightarrow$

$\underline{AD}\ .\wedge \overline{IP} \wedge \overline{SP} \wedge \underline{FL}$

134a. If school disconnectionness with respect to instructional and legitimate affect relations is greater than some value and school passive dependentness with respect to instructional and legitimate affect relations increases and school wholeness with respect to instructional and legitimate affect relations increases, then school supply transmission increases and school spillageness is greater than some value and school regulationness is greater than some value.

134b. $\underline{DC}_{\text{Ins.-Leg.}} \wedge D_P\ \text{Ins.-Leg.}\uparrow \wedge W_{\text{Ins.-Leg.}}\uparrow \Rightarrow$

$FO\uparrow \wedge \underline{SL} \wedge \underline{RG}$

135a. If school disconnectionness with respect to instructional and punishment affect relations is greater than some value and school passive dependentness with respect to instructional and punishment affect relations increases and school wholeness with respect to instructional and punishment affect relations increases and school hierarchically orderness with respect to school instructional and punishment affect relations increases, then if school environmental changeness is greater than some value then school adaptiveness is less than some value, and school supply decreases and school supply transmission decreases and school regulationness decreases and school stableness increases and school equifinalness increases.

135b. $\underline{DC}_{\text{Ins.-Pun.}} \wedge D_P\ \text{Ins.-Pun.}\uparrow \wedge W_{\text{Ins.-Log.}}\uparrow \wedge$

$HO_{\text{Ins.-Pun.}}\uparrow \Rightarrow. \underline{EC}_{\overline{S}} \Rightarrow \overline{AD}\ .\wedge FP\downarrow \wedge FO\downarrow \wedge RG\downarrow \wedge SB\uparrow \wedge EL\uparrow$

4.3. Graph Theoretic Antecedent--Information and Graph Theoretic Consequent

4.3.1. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information and Graph Theoretic Consequent

136a. If maximum school active dependentness with respect to development inquiry and legitimate affect relations, then school supply is less than some value and school filtrationness increases and school spillageness increases and school regulationness is less than some value and school active dependentness with respect to inquiry affect relation decreases and school active dependentness with respect to instructional affect relation increases.

136b. $\max D_{A\ Inq._{Dev.}\text{-}Leg.} \Rightarrow \overline{FP} \wedge FL\uparrow \wedge SL\uparrow \wedge \overline{RG} \wedge D_{A\ Inq.}\downarrow \wedge D_{A\ Ins.}\uparrow$

4.4. Information and Graph Theoretic Antecedent--Graph Theoretic Consequent

137a. If school supply transmission is greater than some value and school compactness is less than some value, then school segregationness is less than some value.

137b. $\underline{FO} \wedge \overline{CO} \Rightarrow \overline{SG}$

4.5. Information and Graph Theoretic Antecedent--Information Theoretic Consequent

138a. If school demand increases and school compactness greater than some value increases, then school regulationness increases.

138b. $TP\uparrow \wedge \underline{CO}\uparrow \Rightarrow RG\uparrow$

139a. If school demand increases and it is not the case that school compactness greater than some value increases, then school efficientness decreases.

139b. $TP\uparrow \wedge \sim\underline{CO}\uparrow \Rightarrow EF\downarrow$

140a. If school supply is constant or school supply decreases and school complete connectionness increases and school strongness increases, then school demand transfer decreases.

140b. $F\vec{P} \vee FP\downarrow \wedge CC\uparrow \wedge SR\uparrow \Rightarrow FT\downarrow$

4.5.1. Information and Graph Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

141a. If school demand increases and school independentness with respect to governing affect relation increases, then school supply transmission increases.

141b. $TP\uparrow \wedge I_{\mathrm{Gov.}}\uparrow \Rightarrow FO\uparrow$

142a. If school supply transfer is greater than some value and school passive dependentness with respect to punishment affect relation and school active dependentness is greater than some value, then school efficientness is greater than some value.

142b. $\underline{FB} \wedge D_{P\ \mathrm{Pun.}} \wedge \underline{D_A} \Rightarrow \underline{EF}$

5. Information and Set Theoretic Hypotheses

5.1. Information Theoretic Antecedent--Set Theoretic Consequent

143a. If school demand transmission is constant, then school homeostasisness is less than some value.

143b. $F\vec{I} \Rightarrow \overline{HS}$

144a. If school filtrationness decreases, then school isomorphismness increases.

144b. $FL\downarrow \Rightarrow IM\uparrow$

145a. If school filtrationness is greater than some value, then school stableness is greater than some value.

145b. $\underline{FL} \Rightarrow \underline{SB}$

146a. If school adaptiveness is greater than some value, then school stableness decreases.

146b. $\underline{AD} \Rightarrow SB\downarrow$

147a. If school demand increases and school supply transmission is nearly minimum, then school stressness increases.

147b. $TP\uparrow \wedge \overline{\overline{\overline{FO}}} \Rightarrow SE\uparrow$

148a. If school environmental changeness is greater than some value and it is not the case that school demand transfer is greater than some value and school supply transfer is greater than some value, then school stableness is less than some value.

148b. $\underline{EC}_{\overline{S}} \wedge \sim\underline{FT} \wedge \underline{FB} \Rightarrow \overline{SB}$

5.1.1. Both Information Theoretic Antecedent and Set Theoretic Consequent with Respect to Affect Relations

149a. If school filtrationness with respect to instructional affect relation increases, then school isomorphismness with respect to instructional affect relation increases.

149b. $FL_{Ins.}\uparrow \Rightarrow IM_{Ins.}\uparrow$

5.2. Set Theoretic Antecedent--Information Theoretic Consequent

150a. If school automorphismness increases, then school resource increases and school storage increases and school supply decreases and school supply transmission decreases and school filtrationness decreases and school spillageness decreases and school efficientness decreases.

150b. $AM\uparrow \Rightarrow IP\uparrow \wedge SP\uparrow \wedge FP\downarrow \wedge FO\downarrow \wedge FL\downarrow \wedge SL\downarrow \wedge EF\downarrow$

151a. If school isomorphismness increases, then school supply decreases and school supply transmission decreases.

151b. $IM\uparrow \Rightarrow FP\downarrow \wedge FO\downarrow$

152a. If school state steadiness is greater than some value, then school adaptiveness is less than some value.

152b. $\underline{SS} \Rightarrow \overline{AD}$

153a. If school state determinationness increases, then school regulationness decreases.

153b. $SD\uparrow \Rightarrow RG\downarrow$

154a. If school state determinationness increases, then school selective informationness decreases.

154b. $SD\uparrow \Rightarrow SI\downarrow$

155a. If school equifinalness is greater than some value, then school regulationness is less than some value.

155b. $\underline{EL} \Rightarrow \overline{RG}$

156a. If school equifinalness at a given time and school homeostasisness is greater than some value, then school regulationness is less than some value.

156b. $EL(t_1) \wedge \underline{HS} \Rightarrow \overline{RG}$

5.2.1. Set Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

157a. If school isomorphismness with respect to instructional affect relation increases, then school supply decreases and school supply transmission decreases.

157b. $IM_{Ins.}\uparrow \Rightarrow FP\downarrow \wedge FO\downarrow$

5.3. Information and Set Theoretic Antecedent--Information Theoretic Consequent

158a. If school demand increases and school sizeness is constant, then school supply transfer increases.

158b. $TP\uparrow \wedge \overrightarrow{SZ} \Rightarrow FB\uparrow$

159a. If school environmental changeness is greater than some value and school compatibleness is greater than some value and school stableness is greater than some value, then school storage is greater than some value or school filtrationness is greater than some value or school spillageness is greater than some value.

159b. $\underline{EC}_{\overline{S}} \wedge \underline{CP} \wedge \underline{SB} \Rightarrow \underline{SP} \vee \underline{FL} \vee \underline{SL}$

160a. If school demand increases and school supply increases and school sizeness is constant, then school supply transmission increases.

160b. $TP\uparrow \wedge FP\uparrow \wedge \overrightarrow{SZ} \Rightarrow FO\uparrow$

161a. If school depletion is constant, and school automorphismness decreases and school homomorphismness is greater than some value, then school supply transmission decreases.

161b. $\overrightarrow{OP} \wedge AM\downarrow \wedge \underline{HM} \Rightarrow FO\downarrow$

5.4. Information and Set Theoretic Antecedent--Set Theoretic Consequent

162a. If school demand is less than some value and school demand transmission increases and school stableness is less than some value, then school stableness increases.

162b. $\overline{TP} \wedge FI\uparrow \wedge \overline{SB} \Rightarrow SB\uparrow$

163a. If school demand is greater than some value and school demand transmission decreases and school stableness is less than some value, then school stableness increases.

163b. $\underline{TP} \wedge FI\downarrow \wedge \overline{SB} \Rightarrow SB\uparrow$

6. Graph and Set Theoretic Hypotheses

6.1. Graph Theoretic Antecedent--Set Theoretic Consequent

164a. If school independentness increases, then school stableness is less than some value.

164b. $I\uparrow \Rightarrow \overline{SB}$

165a. If school flexibleness decreases, then school state determinationness increases.

165b. $F\downarrow \Rightarrow SD\uparrow$

166a. If school centralness increases, then school state steadiness increases.

166b. $CE\uparrow \Rightarrow SS\uparrow$

167a. If school complexness greater than some value increases, then school sizeness increases.

167b. $\underline{CX}\uparrow \Rightarrow SZ\uparrow$

168a. If school independentness increases and school wholeness increases, then school state steadiness is greater than some value.

168b. $I\uparrow \wedge W\uparrow \Rightarrow \underline{SS}$

169a. If school wholeness is greater than some value and school centralness is greater than some value, then school state determinationness is greater than some value.

169b. $\underline{W} \wedge \underline{CE} \Rightarrow \underline{SD}$

6.1.1. Both Graph Theoretic Antecedent and Set Theoretic Consequent with Respect to Affect Relation

170a. If school centralness with respect to instructional affect relation increases, then school isomorphismness with respect to instructional affect relation increases.

170b. $CE_{Ins.}\uparrow \Rightarrow IM_{Ins.}\uparrow$

171a. If school disconnectionness with respect to facilitating affect relation is greater than some value and school wholeness with respect to facilitating affect relation is less than some value, then school state determinationness with respect to facilitating affect relation is less than some value.

171b. $\underline{DC}_{Fac.} \wedge \overline{W}_{Fac.} \Rightarrow \overline{SD}_{Fac.}$

6.2. Set Theoretic Antecedent--Graph Theoretic Consequent

172a. If school automorphismness increases, then school wholeness decreases.

172b. $AM\uparrow \Rightarrow W\downarrow$

173a. If school automorphismness increases, then school centralness decreases.

173b. $AM\uparrow \Rightarrow CE\downarrow$

174a. Change in school sizeness is greater than change in school hierarchically orderness.

174b. $\Delta SZ > \Delta HO$

6.3. Graph Theoretic Antecedent - Graph and Set Theoretic Consequent

175a. If school complexity degenerationness increases, then school size degenerationness increases or school disconnectionness increases.

175b. $XD\uparrow \Rightarrow ZD\uparrow \vee DC\uparrow$

6.4. Set Theoretic Antecedent--Graph and Set Theoretic Consequent

176a. If school state steadiness is less than some value, then school segregationness is less than some value and school integrationness is less than some value and school homeostasisness is less than some value.

176b. $\overline{SS} \Rightarrow \overline{SG} \wedge \overline{IG} \wedge \overline{HS}$

6.5. Graph and Set Theoretic Antecedent--Graph Theoretic Consequent

177a. If maximum school weakness and school sizeness increases, then school passive dependentness increases or school active dependentness increases.

177b. $\max WE \wedge SZ\uparrow \Rightarrow D_P\uparrow \vee D_A\uparrow$

178a. If school hierarchically orderness at a given time is greater than some value and school sizeness at the same time is greater than some value, then school independentness at a later time increases.

178b. $\underline{HO}(t_1) \wedge \underline{SZ}(t_1) \Rightarrow I(t_2)\uparrow$

179a. If school sizeness increases and school complexity growthness is constant, then school vulnerableness increases.

179b. $SZ\uparrow \wedge \vec{XG} \Rightarrow VN\uparrow$

180a. If school sizeness increases and school complexity growthness is constant, then school flexibleness decreases.

180b. $SZ\uparrow \wedge \overrightarrow{XG} \Rightarrow F\downarrow$

181a. If school sizeness increases and school complexity growthness is constant, then school centralness decreases.

181b. $SZ\uparrow \wedge \overrightarrow{XG} \Rightarrow CE\downarrow$

182a. If school sizeness is constant and school complexity degenerationness increases, then school disconnectionness increases.

182b. $\overrightarrow{SZ} \wedge XD\uparrow \Rightarrow DC\uparrow$

183a. If school sizeness decreases and school complexity growthness increases, then school disconnectionness decreases.

183b. $SZ\downarrow \wedge XD\uparrow \Rightarrow DC\downarrow$

184a. If school complexness increases and school size growthness is constant, then school compactness decreases.

184b. $CX\uparrow \wedge \overrightarrow{ZG} \Rightarrow CO\downarrow$

185a. If school complexness increases and school size growthness is constant, then school centralness increases.

185b. $CX\uparrow \wedge \overrightarrow{ZG} \Rightarrow CE\uparrow$

6.6. Graph and Set Theoretic Antecedent--Set Theoretic Consequent

186a. If school centralness increases and school stressness is greater than some value, then school stableness decreases.

186b. $CE\uparrow \wedge \underline{SE} \Rightarrow SB\downarrow$

187a. If school stressness is equal to zero and school centralness increases, then school stableness increases.

187b. $SE = 0 \wedge CE\uparrow \Rightarrow SB\uparrow$

188a. If school sizeness increases and school complexity growthness is constant, then school state determinationness increases.

188b. $SZ\uparrow \wedge \vec{XG} \Rightarrow SD\uparrow$

7. Information, Graph, and Set Theoretic Hypotheses

7.1. Graph Theoretic Antecedent--Information and Set Theoretic Consequent

7.1.1. Graph Theoretic Antecedent with Respect to Affect Relation--Information and Set Theoretic Consequent with Respect to Affect Relation

189a. If maximum school active dependentness with respect to research inquiry and legitimate affect relations, then school resource increases and school supply increases and school storage increases and school filtrationness increases and school automorphismness with respect to instructional affect relation increases.

189b. $\max D_{A\ \mathrm{Inq.}_{\mathrm{Res.}}\text{-}\mathrm{Leg.}} \Rightarrow IP\uparrow \wedge FP\uparrow \wedge SP\uparrow \wedge FL\uparrow \wedge AM_{\mathrm{Ins.}}{}^{\uparrow}$

7.2. Set Theoretic Antecedent--Information, Graph, and Set Theoretic Consequent

190a. If school homomorphismness at a later time is greater than school homomorphismness at a given time, then school demand is nearly maximum and school size degenerationness is nearly maximum and school complexity degenerationness is nearly maximum.

190b. $HM(t_2) > HM(t_1) \Rightarrow \underline{\underline{TP}} \wedge \underline{\underline{ZD}} \wedge \underline{\underline{XD}}$

7.3. Information and Graph Theoretic Antecedent--Set Theoretic Consequent

191a. If school efficientness is greater than some value and school compactness is greater than some value, then school state determinationness is greater than some value.

191b. $\underline{EF} \wedge \underline{CO} \Rightarrow \underline{SD}$

7.4. Information and Set Theoretic Antecedent--Graph Theoretic Consequent

192a. If school size growthness decreases and school selective information growthness is constant, then school complexity growthness increases.

192b. $ZG\downarrow \wedge \overrightarrow{TG} \Rightarrow XG\uparrow$

193a. If school size degenerationness decreases and school selective information degenerationness is constant, then school complexity degenerationness increases.

193b. $ZD\downarrow \wedge \overrightarrow{TD} \Rightarrow XD\uparrow$

7.5. Graph and Set Theoretic Antecedent--Information Theoretic Consequent

194a. If school sizeness increases and school complexity growthness is constant, then school demand increases.

194b. $SZ\uparrow \wedge \overrightarrow{XG} \Rightarrow TP\uparrow$

195a. If school sizeness increases and school complexity growthness is constant, then school demand transmission decreases.

195b. $SZ\uparrow \wedge \overrightarrow{XG} \Rightarrow FI\downarrow$

196a. If school sizeness increases and school complexity growthness is constant, then school supply transmission increases and change in school supply transmission decreases.

196b. $SZ\uparrow \wedge \overrightarrow{XG} \Rightarrow FO\uparrow \wedge \Delta FO\downarrow$

197a. If school sizeness increases and school complexity growthness is constant, then school demand transfer increases.

197b. $SZ\uparrow \wedge \overrightarrow{XG} \Rightarrow FT\uparrow$

198a. If school sizeness increases and school complexity growthness is constant, then school supply transfer decreases.

198b. $SZ\uparrow \wedge \overrightarrow{XG} \Rightarrow FB\downarrow$

199a. If school sizeness increases and school complexity growthness is constant, then school regulationness increases to some value and then decreases.

199b. $SZ\uparrow \wedge X\vec{G} \Rightarrow RG\curvearrowright$

200a. If school sizeness increases and school complexity growthness is constant, then school compatibleness decreases.

200b. $SZ\uparrow \wedge X\vec{G} \Rightarrow CP\downarrow$

201a. If school sizeness increases and school complexity growthness is constant, then school efficientness increases to some value and then decreases.

201b. $SZ\uparrow \wedge X\vec{G} \Rightarrow EF\curvearrowright$

CHAPTER VIII

RELATING THE THEORY TO DATA

Need and Nature of Relating

If nothing can be stated about how the educational theory relates to observations about education (to educational data), then the devising of the theory was simply sheer and idle speculating. It would have been sheer speculating, because there would be no conceivable verification procedure for the resultant theory. It would have been idle speculating, because there would be no use for the resultant theory. A theory that can neither be confirmed nor disconfirmed is one without application. Concepts without percepts are empty.

Something, however, can be stated, even though a complete statement is not possible within the scope of this project. A complete statement would involve the specification of all the decision procedures for relating theory to data (the specification of all the indicators). Since such a specification must be done in the context of data as well as in the context of the theory, a thorough collection of extant educational data is required. This thorough collection could not be undertaken within the limitations of this project but is a part of our next projection with respect to the educational theory.[1] Nevertheless, some specification can be presented, for the theory was not devised apart from more than a cursory examination of extant educational data. Stated differently, the theory was constructed relative to data, and so ways of relating the theory to data are inherent in the theory.

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[1] See the Conclusion.

Presentation of Relating

In the presentation of the relating of the theory to data, paradigm hypotheses of the theory are stated[2] and the specification relative to each cited. That the hypotheses are paradigms is patent from the grouping according to the logico-mathematical structure of the properties related in the hypotheses and the subgrouping according to the similarity of specification within the structural groups.

Group 1: Hypotheses Containing Information Theoretic Properties

Subgroup 1.1: Hypotheses Containing Properties Involving the H-Function--School Demand, TP, School Resource, IP, School Supply, FP, and School Depletion, OP

5a. If school demand increases, then school resource increases to some value and then decreases.

5b. $TP\uparrow \Rightarrow IP\curvearrowright$

5.1. TP and IP can be specified in terms of frequency distribution of components or affect relations of a school's surroundings or of a school respectively relative to a given set of categories.

5.2. Increases, ↑, can be specified as a greater value of the amount of information, H, at one time than at a preceding time. Similarly, decreases, ↓, can be specified as a lesser value of H at one time than at a preceding time. It is patent then how increases to some value and then decreases, ↷, can be specified.

5.3. In this hypothesis and most of the others, implies, ⇒, occurs. The specification for ⇒ is a given decision procedure, e.g. a given statistical procedure, for comparing fit of values (data) with relationships stated in the hypotheses.

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[2] The numbering of the hypotheses in Chapter VII is retained to permit cross referencing.

15a. If school depletion increases, then school supply increases.

15b. $OP\uparrow \Rightarrow FP\uparrow$

15.1. The specification for OP and FP is analogous to that of TP and IP. The categories, however, are not necessarily the same.[3]

Subgroup 1.2: Hypotheses Containing a Property Involving Conditional Distribution--School Storage, SP

37a. If school resource increases and school storage is less than some value, then change in school resource is equal to change in school storage.

37b. $IP\uparrow \wedge \overline{SP} \Rightarrow \Delta IP = \Delta SP$

37.1. Since 'storeputness' was defined in the SIGGS Theory Model as inputness that is not fromputness, the specification of SP is the conditional selective information of IP given FP, $I_S^C(IP|FP)$. The use of this function entails determination of the conditional distribution of IP with respect to FP.

37.2. Is less than some value, $\overline{}$, must be interpreted through an analysis of data which relates to the hypothesis. For example, if such an analysis with respect to $IP\uparrow$, SP, ΔIP, and ΔSP reveals a value, v, for SP such that the hypothesis does not hold beyond v but does hold at values less than v, then v is taken to be the value of $\overline{SP}$.

37.3. Change in, Δ, can be specified qualitatively as two distinct indicated values of a property. One way to specify change quantitatively is to specify a continuum of values for change with respect to a property. To illustrate, the quotient of two values of a property specifies a change continuum which varies from 0 to $+\infty$, and the difference of two values of a property specifies a change continuum which varies from $-\infty$ to $+\infty$. If the values of a property vary in a non-regular fashion, then a more appropriate continuum of change values is specified by considering the difference quotient of that property's values with respect to some regular varying term, e.g. the difference of IP with respect to time:

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[3] When specifications have been cited previously, they are not repeated. In the case of 15b, for instance, the specifications for ↑ and ⇒ were cited previously with respect to 5b.

$$\frac{IP(t_2) - IP(t_1)}{t_2 - t_2}$$

If the limits of this quotient are determined, then the first derivative specifies the continuum.

37.4. When and, $\wedge$, conjoins properties, the interpretation is that values for the conjoined properties are determinable.

37.5. Is equal to, =, is confirmed to be the relation holding between properties, if the values of the properties which '=' connects are equal or do not differ significantly.

Subgroup 1.3: Hypotheses Containing Properties Involving the T-Function--School Demand Transmission, FI, School Supply Transmission, FO, School Demand Transfer, FT, and School Supply Transfer, FB

51a. Change in school demand transmission is greater than change in school supply transmission.

51b. $\Delta FI > \Delta FO$

51.1. FI is specified in terms of the joint distribution of TP and IP, where TP and IP are taken at times t_1 and t_2 respectively and where t_1 precedes t_2. The value of FI then is $T(TP, IP)$, i.e. $T(TP(t_1), IP(t_2))$ where $t_1 < t_2$. Any of the various correlational or multivariate analyses, if the frequency distribution is available, provides the basis for calculating the T-function.

51.2. FO is specified analogously to FI, although the joint distribution involved is that of FP and OP instead of TP and IP.

51.3. Is greater than, >, is specified in the standard way; the value of ΔFI is greater than the value of ΔFO.

33a. If school demand increases and school supply increases, then school demand transfer increases.

33b. $TP\uparrow \wedge FP\uparrow \Rightarrow FT\uparrow$

33.1. The specification of FT is analogous to that of FI and FO. The joint distribution, however, involves TP, IP, FP, and OP where TP, IP, FP, and OP are taken at times t_1, t_2, t_3, and t_4 respectively and where t_1 precedes t_2, t_2 precedes t_3, and t_3 precedes t_4.

24a. If school supply transfer is greater than some value, then school storage is less than some value.

24b. $\underline{FB} \Rightarrow \overline{SP}$

24.1. The specification of FB is analogous to that of FT. The difference is that TP, IP, FP, and OP are not taken at the same times. FP, OP, TP, and IP are taken at times t_1, t_2, t_3, and t_4 respectively, where t_1 precedes t_2, t_2 precedes t_3, and t_3 precedes t_4.

24.2. Is greater than some value, $\underline{\quad}$, is specified analogously to $\overline{\quad}$.

Subgroup 1.4: Hypotheses Containing Properties Involving the B-Function--School Compatibleness, CP, and School Efficientness, EF

21a. If school demand transfer increases, then school compatibleness increases.

21b. $FT\uparrow \Rightarrow CP\uparrow$

21.1. 'CP' is defined as commonality between FI and FO, i.e. B(FI,FO). The use of B(FI,FO) as a specification for CP depends upon the specification of the distributions underlying FI and FO which are joint distributions of TP and IP, and FP and OP respectively, and the specification of the joint distributions of TP, IP, FP, and OP. When these joint distributions are determined, the B-function is calculable.

41a. If school resource is constant and school efficientness at a given time is less than some value, then school efficientness increases.

41b. $\vec{IP} \wedge \overline{EF(t_1)} \Rightarrow EF\uparrow$

41.1. EF, being defined as B(FT,TP), requires the determination of the distributions underlying FT and TP.

41.2. When a property is expressed as a function of time, the unit of time over which the values for the property are determined must be specified. Hence, the confirmation of the hypothesis depends on the selection of the time interval.

41.3. Is constant, $\rightarrow$, is specified in the standard way; the value of IP does not change.

Subgroup 1.5: Hypotheses Containing a Property Involving Property States--School Openness, O

29a. If school openness increases, then school efficientness decreases.

29b. $O\uparrow \Rightarrow EF\downarrow$

29.1. O is structured in terms of property states: $ST_{FI} + ST_{FO} - ST_{CP}$. Consequently, upon the specification of FI, FO, and CP, the relating of values in the prescribed way yields the value of O.

Subgroup 1.6: Hypotheses Containing Properties Involving Max--School Filtrationness, FL, and School Spillageness, SL

26a. If school filtrationness is greater than some value, then school compatibleness is greater than some value.

26b. $\underline{FL} \Rightarrow \underline{CP}$

26.1. The formal requirement for filtrationness is that it be a differential between maximum toputness state, max ST_{TP}, and toputness state, ST_{TP}. Specification for FL, therefore, depends upon specification of TP which has been discussed, specification of max TP, and specification of the differential between max TP and TP. Max TP can be specified as the maximum number of school demand categories possible for a school. Another specification is given the range of possible school demand categories, TP′, the value of max TP, max ST_{TP}, is max H(TP′), i.e. the case when alternatives are equiprobable. The differential between max TP and TP can be specified as the difference between the indicated values of max TP and TP.

39a. If school resource is greater than some value and school spillageness is less than some value, then school storage increases.

39b. $\underline{IP} \wedge \overline{SL} \Rightarrow SP\uparrow$

39.1. The formal requirement for spillageness is that it be a differential between maximum feedinness state, max ST_{FI}, and feedinness state, ST_{FI}. Specification for SL, therefore, depends upon specification of FI which has been discussed, specification of max FI, and specification of the differential between max FI and FI. The latter two specifications are analogous to those with respect to FL.

Subgroup 1.7: Hypotheses Containing a Property Involving Amount of Selective Information--School Selective Informationness, SI

42a. If the ratio of maximum school selective informationness to school resource decreases, then school supply transmission decreases.

42b. $\frac{\max SI}{IP}\downarrow \Rightarrow FO\downarrow$

42.1. The specification of SI requires a classification of informations which are in a school.

42.2. The ratio is to be interpreted as the quotient of the property values which it relates.

Group 2: Hypotheses Containing Graph Theoretic Properties

Subgroup 2.1: Hypotheses Containing Properties Involving Affect Relation Configurations--School Complete Connectionness, CC, School Strongness, SR, School Unilateralness, U, School Weakness, WE, School Compactness, CO, and School Centralness, CE

106a. If school complete connectionness increases or school strongness increases, then school demand increases.

106b. $CC\uparrow \vee SR\uparrow \Rightarrow TP\uparrow$

106.1. CC can be specified in terms of two way connections between school components, since 'CC' is defined as every two components directly channeled to each other with respect to affect relations.

106.2. If any specification is to be given to CC↑, then change in CC must be specified, i.e. a range of values for CC. The ratio

$$\frac{\text{number of two way direct channels}}{\text{max number of two way direct channels}}$$

specifies a discrete set of values which ranges from 0 (no two way connections) through various fractions of CC to 1 (all possible two way direct channels). Such a specification makes values of CC independent of school sizeness, SZ, because the max number of two way direct channels provides a relational not a size basis for the scale. Thus, a CC value of $\frac{1}{2}$ means that, independently of SZ, $\frac{1}{2}$ of the possible two way connections are present. An increase in CC, then, is taken to be an increase in the ratio. It is to be noted that the max number of two way direct channels is determinable as a function of sizeness, i.e.

$$\frac{n(n - 1)}{2}$$

where n is the number of components.

106.3. The specification for SR is analogous to that given for CC, except that the range of SR values is determined by the ratio

$$\frac{\text{number of cycles}}{\text{max number of cycles}}$$

The rationale for the ratio is that the mathematical existence of strong connections between components is a necessary and sufficient condition for the mathematical existence of cycles containing these components.

106.4. When either ... or, v, joins properties, the interpretation is that values for any of the properties or any combination of them are determinable.

91a. If school demand transmission decreases, then school unilateralness decreases.

91b. $FI\downarrow \Rightarrow U\downarrow$

91.1. The range of U values is determined by the ratio

$$\frac{\text{number of one way directed channels}}{\text{max number of one way directed channels}}$$

95a. If school demand transfer increases, then school weakness is less than some value.

95b. $FT\uparrow \Rightarrow \overline{WE}$

95.1. Following the formal requirements of the SIGGS Theory Model, WE is a kind of connectionness which is not either CC or SR or U, and so the specifications for CC, SR, and U can be used to specify WE. For example, $\overline{WE}$ is determined when there are no unconnected cchool components and there are no instances of either $\underline{U}$ or $\underline{SR}$ or $\underline{CC}$.

104a. If school compactness greater than some value increases, then school efficientness increases.

104b. $\underline{CO}\uparrow \Rightarrow EF\uparrow$

104.1. Through graph theory, the average number of direct channels in a channel between school components is explicated formally as

$$p = \sum_{\substack{k=1 \\ m=1 \\ k\neq m}}^{n} \frac{d(s_i,s_j) - d(s_k,s_m)}{n^2 - n}$$

Analysis shows that s_i and s_j are two school components such that the minimum member of direct channels between them (represented by $d(s_i,s_j)$) is greater than or equal to the minimum number of direct channels between any two other school components s_k and s_m (represented by $d(s_k,s_m)$). Hence, the difference $d(s_i,s_j) - d(s_k,s_m)$ decreases as $d(s_k,s_m)$ approximates $d(s_i,s_j)$ and since there are $n^2 - n$ pairs of distinct school components (n^2 pairs of school components - n pairs where a school component is paired with itself), p represents the approximation of average distance to maximum distance ($d(s_i,s_j)$). If this approximation is high, the value p is low, i.e. if all the distances between school components are nearly maximum, CO is low. Conversely, if the ratio of short channels to long channels is high, CO is high. Moreover, CO increases with maximum channel length which is another basis for choosing p as a measure of CO.

69a. If school centralness is less than some value, then school centralness increases.

69b. $\overline{CE} \Rightarrow CE\uparrow$

69.1. The formal requirements for specification of CE are less explicit than for CO, but there is similarity in the structure of both definitions. Thus, a set of school components, A[4], is sorted out such that the set of all channels emanating from A, $\Delta_{oR_{DA}}(A)$, contains the set of all channels emanating from any other set of components $\Delta_{oR_{DA}}(B)$. A measure such as

$$\sum_{B\in S} n(\Delta_{oR_{DA}}(A)) - n(\Delta_{oR_{DA}}(B))$$

where $n(\Delta_{oR_{DA}}(A))$ is the number of channels in $\Delta_{oR_{DA}}(A)$ is similar to p in 104.1 on p. 176, increasing both with the differential between $\Delta_{oR_{DA}}(A)$ and $\Delta_{oR_{DA}}(B)$ and with the magnitude of $\Delta_{oR_{DA}}(A)$.

Subgroup 2.2: Hypotheses Containing Properties Involving Component Configurations--School Passive Dependentness, D_P, School Active Dependentness, D_A, School Independentness, I, School Interdependentness, ID, School Wholeness, W, School Flexibleness, F, School Disconnectionness, DC, and School Vulnerableness, VN

61a. If school passive dependentness increases, then school centralness increases.

61b. $D_P\uparrow \Rightarrow CE\uparrow$

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[4] In this and subsequent definitions, when some subset of the school group is sorted out, it is to be noted that the school components of the subset are not necessarily related to one another.

61.1. A school has the property of D_P when every school component in some subset, A, of the school has channels to it. Therefore, $D_P\uparrow$ is specified to be an increase in the number of school components in A.

62a. If school active dependentness increases, then school centralness decreases.

62b. $D_A\uparrow \Rightarrow CE\downarrow$

62.1. The specification for $D_A\uparrow$ is analogous to $D_P\uparrow$. Modification would arise, of course, in that the channels are from rather than to the subset.

68a. If school centralness is less than some value, then school independentness increases.

68b. $\overline{CE} \Rightarrow I\uparrow$

68.1. A school has the property of I when every school component in some subset, A, of the school has no channels to it. Therefore, the specification for $I\uparrow$ is an increase in the number of school components in A.

102a. If school interdependentness increases, then school demand transmission increases.

102b. $ID\uparrow \Rightarrow FI\uparrow$

102.1. The specification for $ID\uparrow$ is analogous to $I\uparrow$. Modification would arise, since the channels are to and from rather than not to A.

54a. If school strongness decreases, then school wholeness increases.

54b. $SR\downarrow \Rightarrow W\uparrow$

54.1. A school has the property of W when every school component in some subset, A, of the school has channels to every other school component of the entire school. Max W is taken to be the case in which only one school component, s, has channels to every other school component, i.e. $A = \{s\}$. The number of school components in A is not taken as a specification of $W\uparrow$ but of $W\downarrow$. If $n(A)$ represents the number of school components in A, then $\frac{1}{n(A)}$ specifies a set of values which increases from 1 to $+\infty$ which is appropriate for specifying $W\uparrow$.

56a. If school strongness increases, then school flexibleness increases.

56b. $SR\uparrow \Rightarrow F\uparrow$

56.1. Since 'F' is defined as different subgroups of school components through which there is a channel between two school components with respect to affect relations, one way to specify a range of values which indicates increasing or decreasing F is to specify F as the number of distinct pairs of school components connected by at least two channels defined through non-overlapping sets of school components. F↑, then, would be specified as an increase in the number of such distinct pairs.

97a. If school demand transmission increases and school compatibleness is nearly minimum, then school disconnectionness increases.

97b. $FI\uparrow \wedge \overline{\overline{CP}} \Rightarrow DC\uparrow$

97.1. The specification of DC↑ is an increase in the number of sets of school components such that the school components in each set are connected but there are no connections between components in different sets.

97.2. Is nearly minimum, $\overline{\overline{}}$, must be interpreted through an analysis of data which relates to the hypothesis. Such an interpretation is analogous to $\overline{}$ as set forth in 37.2 on p. 170.

60a. If school vulnerableness increases, then school complete connectionness decreases.

60b. $VN\uparrow \Rightarrow CC\downarrow$

60.1. VN↑ is specified as an increase in the number of school components which when removed produce DC.

Subgroup 2.3: Hypotheses Containing a Property Involving Levels Within the Affect Relation Configuration--School Hierarchically Orderness, HO

64a. If school hierarchically orderness increases, then school vulnerableness increases and school flexibleness decreases.

64b. $HO\uparrow \Rightarrow VN\uparrow \wedge F\downarrow$

64.1. The specification of $HO\uparrow$ is an increase in the number of levels of subordinateness as these levels are set forth in the logico-mathematical definition of 'HO'.

Subgroup 2.4: Hypotheses Containing a Property Involving the Number of Connections with Respect to Affect Relations--School Complexness, CX

125a. If school complexness with respect to facilitating affect relations is greater than some value, then school demand transfer with respect to facilitating affect relation is less than some value.

125b. $\underline{CX}_{Fac.} \Rightarrow \overline{FT}_{Fac.}$

125.1. The specification of $CX_{Fac.}$ is the number of connections in the facilitating affect relation of a school.

Group 3: Hypotheses Containing Set Theoretic Properties

Subgroup 3.1: Hypotheses Containing a Property Involving the Number of Components in a Group--School Sizeness, SZ

167a. If school complexness greater than some value increases, then school sizeness increases.

167b. $\underline{CX}\uparrow \Rightarrow SZ\uparrow$

167.1. The specification of $SZ\uparrow$ is an increase in the number of school components.

Subgroup 3.2: Hypotheses Containing Properties Involving Similarity of Structure as Determined by Set Theoretic Mappings--School Automorphismness, AM, School Isomorphismness, IM, and School Homomorphismness, HM

172a. If school automorphismness increases, then school wholeness decreases.

172b. $AM\uparrow \Rightarrow W\downarrow$

172.1. For the formal explication of automorphism, see p. 8, number 20. Stated less formally, an automorphic mapping is a one to one mapping of a set onto itself such that all relations are preserved. Stated metaphorically, a checkerboard can be used without distinguishing between the sides of the board used by a particular player; that is, although one can distinguish between the sides of the board by numbering the squares (the side numbered 1 through 32 always could be distinguished from the side numbered 33 through 64), the relation of the red squares to the black squares and the operations performed on them (moves) are identical whether or not the board is rotated 180° before the game begins. An automorphic mapping of the checkerboard into itself exists, therefore, which takes a square into the square on the opposite side which will occupy the position of the first square after the board is rotated. The number of automorphic mappings possible for a school affords a measure of the degree to which school components bear the same structural relationship to other school components, i.e. the degree of "democratic structuredness".

172.2. The specification of $AM\uparrow$ is an increase in the number of automorphic mappings.

157a. If school isomorphismness with respect to instructional affect relation increases, then school supply decreases and school supply transmission decreases.

157b. $IM_{Ins.}\uparrow \Rightarrow FP\downarrow \wedge FO\downarrow$

157.1. For the formal explication of isomorphism, see p. 8, number 19. Stated less formally, an isomorphic mapping is a one to one mapping of a set onto another set. Utilizing the metaphor again, it makes no difference in the game of checkers whether one checkerboard or another is used. An isomorphic mapping exists between checkerboards. Although the formal requirements permit overlapping of the sets being mapped, the hypotheses are claimed only between non-overlapping sets of school components, in the case of 157a and 157b between non-overlapping sets of instructional components.

157.2. The specification for $IM_{Ins.}\uparrow$ is an increase in the number of distinct instructional subsets which can be isomorphically mapped onto one another.

161a. If school depletion is constant and school automorphismness decreases and school homomorphismness is greater than some value, then school supply transmission decreases.

161b. $\overrightarrow{OP} \wedge AM\downarrow \wedge \underline{HM} \Rightarrow FO\downarrow$

161.1. For a formal explication of homomorphism, see p. 7, number 18. It follows from this explication that through homomorphic mappings the structure on a set can be compared with the structure on a simplified set. A very general specification of HM is as follows: a school can be partitioned into non-overlapping subsets and the degree to which the relations between school components can be represented by relations between the subsets of which these components are elements corresponds to the degree of HM.

Subgroup 3.3: Hypotheses Containing Properties Involving Conditions on a School--School Stableness, SB, School State Determinationness, SD, and School Equifinalness, EL

163a. If school demand is greater than some value and school demand transmission decreases and school stableness is less than some value, then school stableness increases.

163b. $\underline{TP} \land FI\downarrow \land \overline{SB} \Rightarrow SB\uparrow$

163.1. SB is no change with respect to school conditions, i.e. the state characterizing a school at a given time. Therefore, SB↑ is specified as an increase in the number of non-varying property specifications (values) over a given time interval.

165a. If school flexibleness decreases, then school state determinationness increases.

165b. $F\downarrow \Rightarrow SD\uparrow$

165.1. SD is derivability of conditions from one and only one state. In formal terms, there is some subset, A, of a school state such that some other school state uniquely determines A. One way to specify SD is to select a school type of a given state, i.e. having given property specifications. If at a later time another state characterizes all of the schools of that type, then SD obtains.

165.2. Given values of F which are decreasing, the specification of SD↑ would depend upon an increase in the number of consequent property specifications in A.

155a. If school equifinalness is greater than some value, then school regulationness is less than some value.

155b. $\underline{EL} \Rightarrow \overline{RG}$

155.1. EL is derivability of conditions from other states. EL is an opposite to SD in that for some subset, A, of a school state there are a number of distinct school states which determine A. If a specification for EL analogous to SD were used, then EL would obtain provided schools having different property specifications are in the same state at a later time.

155.2. The specification for RG is stated in 34.1. on p. 184.

Group 4: Hypotheses Containing Properties Characterizing Rate of Change of Some Other Property

Subgroup 4.1: Hypothesis Containing Properties Involving Rate of Change of Information Theoretic Properties--School Environmental Changeness, $EC_{\overline{S}}$, School Regulationness, RG, and School Adaptiveness, AD

1a. If school environmental changeness increases, then change in school resource is greater than some value.

1b. $EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta IP}$

1.1. '$EC_{\overline{S}}$' is defined as a difference in school environmentness, $E_{\overline{S}}$. The formal condition for $EC_{\overline{S}}$ to hold is

$$|ST_{E_{\overline{S}}}(t + \Delta t) - ST_{E_{\overline{S}}}(t)| \geqq \delta$$

i.e. the school environment state varies over some time interval, Δt, within certain limits, δ. Since 'a property state' is defined as a property's value at a given time, the specification for $EC_{\overline{S}}$ depends upon the specification for school environmentness, $E_{\overline{S}}$, and hence on school demand, TP, ('TP' is defined as $E_{\overline{S}}$). The value which the difference in TP values never falls below is the value of δ or $EC_{\overline{S}}$. The absolute value signs used in the definition specify that it is the interval within which the TP values are contained, and not whether the difference is positive or negative, which is significant.

34a. If school demand is constant and school efficientness is greater than some value, then school regulationness is less than some value.

34b. $\overrightarrow{TP} \wedge \underline{EF} \Rightarrow \overline{RG}$

34.1. 'RG' is defined as adjustment of FP. The formal condition for RG to hold is

$$|ST_{FP}(t + \Delta t) - ST_{FP}(t)| \geqq \delta$$

i.e. that school supply state varies over some time interval, Δt, within certain limits, δ. The specification of RG depends upon FP just as the specification of $EC_{\overline{S}}$ depends upon TP.

146a. If school adaptiveness is greater than some value, then school stableness decreases.

146b. $\underline{AD} \Rightarrow SB\downarrow$

146.1. AD is a difference in school compatibleness under school environmental changeness:

$$|ST_{CP}(t + \Delta t) - ST_{CP}(t)| \geqq \delta \land EC_{\overline{S}}$$

The specification of AD depends upon CP just as the specification of $EC_{\overline{S}}$ depends upon TP. However, $EC_{\overline{S}}$ must be specified also.

Subgroup 4.2: Hypotheses Containing Properties Involving Rate of Change of Graph Theoretic Properties--School Integrationness, SI, and School Segregationness, IG

98a. If school storage increases and school filtrationness decreases or school spillageness decreases, then school integrationness increases.

98b. $SP\uparrow \land .\ FL\downarrow \lor SL\downarrow\ .\Rightarrow IG\uparrow$

98.1. IG is school wholeness under school environmental change:

$$|ST_{W}(t + \Delta t) - ST_{W}(t)| \leqq \delta \land EC_{\overline{S}}$$

i.e. school wholeness state varies over some time interval, Δt, within certain limits, δ. The dependence of specification for IG on that of W is as follows: the value which the difference in W values never exceeds is the value of δ.

176a. If school state steadiness is less than some value, then school segregationness is less than some value and school integrationness is less than some value and school homeostasisness is less than some value.

176b. $\overline{SS} \Rightarrow \overline{SG} \wedge \overline{IG} \wedge \overline{HS}$

176.1. SG is school independentness under school environmental change:

$$|ST_I(t + \Delta t) - ST_I(t)| \leqq \delta \wedge EC_{\overline{S}}$$

i.e. school independentness state varies over some time interval, Δt, within certain limits, δ. The specification of SG is analogous to IG.

Subgroup 4.3: Hypotheses Containing Properties Involving Rate of Change of Set Theoretic Properties--School State Steadiness, SS, School Homeostasisness, HS, School Stressness, SE, and School Strainness, SA

176.2. SS is school stableness under school environmental change:

$$|ST_{SB}(t + \Delta t) - ST_{SB}(t)| \leqq \delta \wedge EC_{\overline{S}}$$

i.e. school stableness state varies over some time interval, Δt, within certain limits, δ. The specification of SS is analogous to IG.

176.3. HS is school equifinalness under school environmental change:

$$|ST_{EL}(t + \Delta t) - ST_{EL}(t)| \leqq \delta \wedge EC_{\overline{S}}$$

i.e. school equifinalness state varies over some time interval, Δt, within certain limits, δ. The specification of HS is analogous to IG.

186a. If school centralness increases and school stressness is greater than some value, then school stableness decreases.

186b. $CE\uparrow \wedge \underline{SE} \Rightarrow SB\downarrow$

186.1. SE is change beyond certain limits of school's surroundings state:

$$|ST_{\overline{S}}(t + \Delta t) - ST_{\overline{S}}(t)| \geqq \delta$$

The relation between the specification of SE and $\overline{\delta}$ is the same as that between $EC_{\overline{S}}$ and TP in 1.1 on p. 183.

86a. If school state steadiness is greater than some value, then school strainness increases.

86b. $\underline{SS} \Rightarrow SA\uparrow$

86.1. SA is change beyond certain limits of school state:

$$|ST_{\overline{S}}(t + \Delta t) - ST_{\overline{S}}(t)| \geqq \delta$$

The specification of SA is analogous to SE.

Group 5: Hypotheses Containing Properties Characterizing Change of Some Other Property

Subgroup 5.1: Hypotheses Containing Properties Involving Increase of Some Other Property--School Size Growthness, ZG, School Complexity Growthness, XG, and School Selective Information Growthness, TG

192a. If school size growthness decreases and school selective information growthness is constant, then school complexity growthness increases.

192b. $ZG\downarrow \wedge \overrightarrow{TG} \Rightarrow XG\uparrow$

192.1. ZG is increase in school sizeness:

$$ST_{SZ}(t + \Delta t) \geqq ST_{SZ}(t)$$

i.e. once values for SZ over a time interval are determined, the value of ZG is some measure of an increment in size. Similar statements can be made for XG and TG.

Subgroup 5.2: Hypotheses Containing Properties Involving Decrease of Some Other Property--School Size Degenerationness, ZD, School Complexity Degenerationness, XD, and School Selective Information Degenerationness, TD

193a. If school size degenerationness decreases and school selective information degenerationness is constant, then school complexity degenerationness increases.

193b. $ZD\downarrow \wedge \overrightarrow{TD} \Rightarrow XD\uparrow$

193.1. ZD is decrease in school sizeness:

$$ST_{SZ}(t + \Delta t) \leqq ST_{SZ}(t)$$

i.e. once values for SZ over a time interval are determined, the value of ZD is some measure of a decrement in size. Similar statements can be made for XD and TD.

CONCLUSION

The conclusion of this report depends upon the projection of another activity with respect to the educational theory which has been developed in this project. Unless the adequacy of the educational theory is known, its knowledge status is problemmatic. The only legitimate conclusion, therefore, is a projection of what must be done to determine the adequacy of the theory, i.e. to evaluate the theory.

Evaluation of an empirical theory, such as the educational theory presented in this report, consists not only in testing the theory but also in setting forth its predictive power. Moreover, evaluation of a theory involves one in a concomitant activity, modifying the theory. When inadequacies in a theory become known, modification of the theory to increase its adequacy becomes possible. Since evaluation involves testing, setting forth predictive power, and modifying, these general procedures must be clarified. In testing the theory, what is required is to estimate the fit between the hypotheses constituting the theory and the educational data. The fit is estimated through indicators, i.e. through decision procedures for relating the theory to data. In setting forth the predictive power of the theory, what is required is delineation of the hypotheses for which there is no data but for which data could be found. The data gaps only can be determiners of predictive power provided indicators can be specified for the finding of the data. In modifying the theory, what is required is to change or extend it in terms of formation or transformation rules in order to secure fit or predictive power. A formation rule is a decision procedure for the syntax of a hypothesis, i.e. for putting together a hypothesis; while a transformation rule is one for the syntax of a group of hypotheses, i.e. for deriving one or more hypotheses from one or more other hypotheses.

To increase the specificity of this projection and so of this conclusion, two outlines of the tasks to be carried out are presented: on page 190 a verbal outline which lists them, and on page 191 a schematic outline which exhibits their interrelations and their results.

It is patent from the outlines that a computer will be utilized in the projected evaluation and modification of the educational theory, in spite of the fact that it has been neglected in education[1] for such use. Articles by Baker[2] and Goodlad[3] substantiate this neglect as did a literature survey based upon a review of citations in the Education Index, the Index to Periodical Literature, and the Review of Educational Research and of projects contracted by the U. S. Office of Education. Baker attributes this neglect to the state of educational theory:

> The nebulous theories prevalent in the educational world cannot survive the cold realities of programming for a digital computer.[4]

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[1] In other fields, the computer has shown great promise. See The Use of Computers, edited by Dell Hymes, The Hague: Mouten & Co., 1965.

[2] Frank B. Baker, "The Use of Computers in Educational Research," Review of Educational Research, Vol. 33, 1963, pp. 566-578.

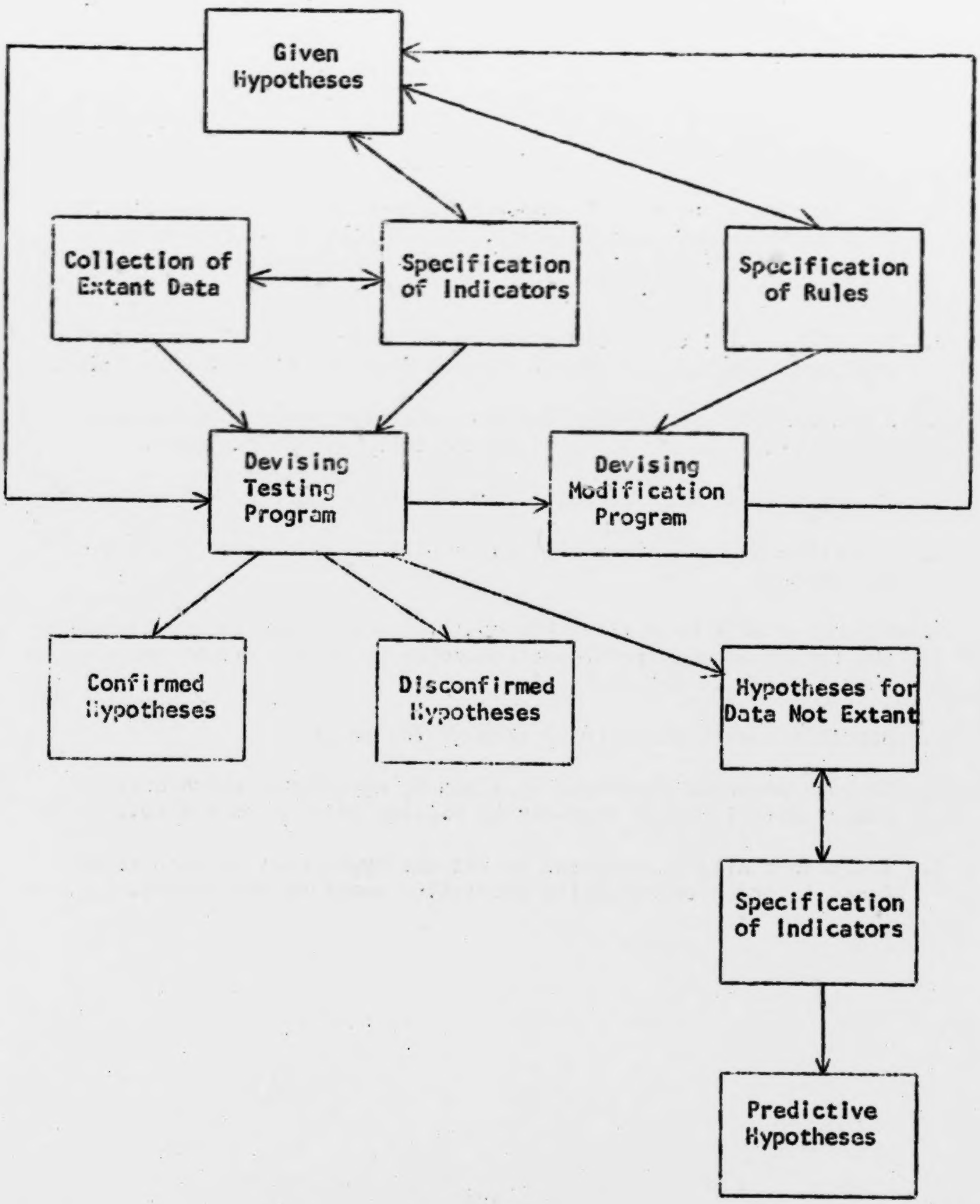
[3] John I. Goodlad, et. al. Applications of Electronic Data Processing Methods in Education, Washington, D. C.: Office of Education, Department of Health, Education, and Welfare, Cooperative Research Project No. F-026, 1965.

[4] Baker, op. cit., p. 573.

Verbal Outline

1. Extant data about school components, operations, and organization will be collected from agencies, such as the U. S. Office of Education, regional, state, and local administrative bureaus, and published reports of educational research.
2. Indicators will be specified in terms of the hypotheses of the empirical educational theory derived from the SIGGS Theory Model.
3. A program will be devised for determination through indicators of the fit of the hypotheses to the collected extant data.
4. The hypotheses will be tested through the program.
5. Formation and transformation rules will be specified in terms of the theory.
6. A program will be devised for modification of the theory through the formation and transformation rules to secure fit of those hypotheses which do not test out.
7. Hypotheses will be modified through the program.
8. Through programs developed in 3 and 6, hypotheses which have no extant data to which they can be related will be sorted out.
9. Indicators will be proposed to fit the hypotheses to data to be found in order to establish predictive power of the theory.

Schematic Outline


Given Hypotheses
Collection of Extant Data
Specification of Indicators
Specification of Rules
Devising Testing Program
Devising Modification Program
Confirmed Hypotheses
Disconfirmed Hypotheses
Hypotheses for Data Not Extant
Specification of Indicators
Predictive Hypotheses

The development of the educational theory from set theory (S), information theory (I), graph theory (G), and general systems theory (GS) integrated into the SIGGS Theory Model has set the stage for survival.

APPENDICES

Appendix I

Predicate Calculus and the Model

Since the model includes terms which are defined, predicate calculus enters into the model to indicate the definiendum-definens relation. In terms of predicate calculus

$$\ldots =_{Df} --- \qquad \text{(Appendix II, 1)}$$

is interpreted as

$$\forall x(\ldots \Leftrightarrow ---x)$$

which is read as

"for all x, x is a ... if and only if x is a ---"

'$=_{Df}$' is utilized to simplify the presentation. For example, the logico-mathematical definition of group, S, in the model is

$$S =_{Df} \{s_i \mid 1 \leq i \wedge i \leqq n \wedge n \geqq 2\} \qquad \text{(Chapter IV, 3.2)}$$

instead of

$$\forall x(Sx \Leftrightarrow \{s_i \mid 1 \leq i \wedge i \leqq n \wedge n \geqq 2\}x)$$

Universal quantifiers

$$\forall \ldots(---) \qquad \text{(Appendix II, 19)}$$

other than those involved in $\ldots =_{Df} ---$ are specified.

Descriptive quantifiers

$$\iota \ldots(---) \qquad \text{(Appendix II, 8)}$$

are specified except with respect to mathematical entities, such as sets.

Existential quantifiers

$$\exists \ldots(---) \qquad \text{(Appendix II, 10)}$$

are not specified except where it was thought greater clarity would result. The elimination of descriptive and existential quantifiers also is done to produce more simplification of presentation.

One final simplification procedure is the elimination of the marks for cross-referencing quantifiers. For example, the logico-mathematical definition of system, $\overline{S}$, in the model is

$$\begin{aligned}&\overline{S} =_{Df} S \mid \exists \mathcal{R}_A(\mathcal{R}_A \neq \emptyset \wedge \forall R_A(R_A \in \mathcal{R}_A \Rightarrow. R_A \subset S \times S \wedge \\ &\exists \mathcal{I}(\mathcal{I} \neq \emptyset \wedge \forall I(I \in \mathcal{I} \Rightarrow. I \sim R_A \;\overline{\vee}\; \exists \mathcal{R}(\mathcal{R} \subset R_A^{2^n} \wedge \\ &I \sim \mathcal{R}) \;\overline{\vee}\; \exists S'(S' \subset S \wedge I \sim S'))))) \qquad \text{(Chapter IV, 8.2)}\end{aligned}$$

Introducing the marks for cross-referencing the quantifiers, the definition becomes

$$\begin{aligned}&\forall t(\overline{S}t \Leftrightarrow St \wedge \exists u((\mathcal{R}_A)u \wedge u \neq \emptyset \wedge \forall v((R_A)v \Rightarrow. v \in u \Rightarrow v \subset t \times t \wedge \\ &\exists w(\mathcal{I}w \wedge w \neq \emptyset \wedge \forall x(Ix \Rightarrow: x \in w \Rightarrow x \sim v \;.\overline{\vee}\; \exists y(\mathcal{R}y \wedge y \subset v^{2^n} \wedge \\ &x \sim y) \;\overline{\vee}\; \exists z(S'z \wedge z \subset t \wedge x \sim z))))))\end{aligned}$$

Appendix II

Translation of Syntactical Symbols

Logico-mathematical Symbols	Verbal Symbols
1. ... $=_{Df}$ ---	... equals by definition ---
2. $\{...\}$	set of elements ...
3. ... $\mid$ ---	... such that ---
4. ... $\leqq$ ---	... is less than or equal to ---
5. ... $\wedge$ ---	... and ---
6. ... $\geqq$ ---	... is greater than or equal to ---
7. ... = ---	... is equal to ---
8. ι...(---)	that ... such that ---
9. ... $\in$ ---	... is an element of ---
10. $\exists$...(---)	there is a ... such that ---
11. ... < ---	... is less than ---
12. (...,---,n)	n-tuple of ... and --- and n
13. ...(---)	... at ---
14. ... < ---	... precedes ---
15. ... + ---	... plus ---
16. ... $\subset$ ---	... is contained in ---
17. ... × ---	Cartesian product of ... and ---
18. ... $\neq$ ---	... is not equal to ---
19. $\forall$...(---)	for all ..., ---
20. ... $\Rightarrow$ ---	... only if ---

Logico-mathematical Symbols	Verbal Symbols
21. ... $\notin$ ---	... is not an element of ---
22. ... $\sim$ ---	... is equivalent to ---
23. ... ∇ ---	either ... or --- and not both
24. ...$^{2^{n}}$	power set of ...
25. $C_{\ldots}$---	complement of --- with respect to ...
26. n(...)	cardinality of ...
27. |...|	absolute value of ...
28. Δ...	increment of ...
29. ... - ---	... minus ---
30. max ...	maximum ...
31. $\bigcup_{i=1}^{n} \ldots_i$	union of ... where ... is indexed from 1 to n
32. ... $\cup$ ---	union of ... and ---
33. $\bigwedge_{i=1}^{n} \ldots_i$	conjunction of ... where ... is indexed from 1 to n
34. ... $\cap$ ---	intersection of ... and ---
35. ... > ---	... is greater than ---
36. ... $\rightarrow$ ---	... into ---
37. $\sum_{i=1}^{n} \ldots_i$	summation of ... where ... is indexed from 1 to n
38. $\bigcup_{c \in \text{---}} \ldots_c$	union of ... as c varies over ---
39. $\sum_{c \in \text{---}} \ldots_c$	summation of ... as c varies over ---
40. ... $\vdash$ ---	... yields ---

Appendix III

List of Hypotheses According to the Sequencing of Properties in the SIGGS Theory Model[1]

20. $EC_{\overline{S}}$: $EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta IP}$ 1

$EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FP}$ 2

$EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FB}$ 3

$EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FL}$ 4

$EC_{\overline{S}}\uparrow \wedge FP\uparrow \Rightarrow \underline{\Delta FO}$ 30

$EC_{\overline{S}}\uparrow \wedge FP\uparrow \Rightarrow \underline{\Delta FT}$ 31

$\underline{EC_{\overline{S}}} \wedge \underline{FT} \Rightarrow \underline{SB}$ 32

$\underline{EC_{\overline{S}}} \wedge \sim\underline{FT} \wedge \underline{FB} \Rightarrow \overline{SB}$ 148

$\underline{EC_{\overline{S}}} \wedge \underline{CP} \wedge \underline{SB} \Rightarrow \underline{SP} \vee \underline{FL} \vee \underline{SL}$ 159

22. TP: $TP\uparrow \Rightarrow IP\circlearrowleft$ 5

$\underline{TP}\uparrow \Rightarrow FP\uparrow$ 6

$\overline{\overline{TP}} \Rightarrow FP\uparrow$ 7

$TP\uparrow \Rightarrow FL\circlearrowleft$ 8

$TP\uparrow \Rightarrow \overline{RG}\uparrow$ 9

$TP\uparrow \Rightarrow CE\downarrow$ 90

$TP\uparrow \wedge FP\uparrow \Rightarrow FT\uparrow$ 33

$\overline{\overline{TP}} \wedge FP\uparrow \Rightarrow DC\uparrow$ 96

$TP\uparrow \wedge \overline{\overline{FO}} \Rightarrow SE\uparrow$ 147

$T\vec{P} \wedge \underline{EF} \Rightarrow \overline{RG}$ 34

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[1] To permit cross referencing, the numbering of the properties and of the hypotheses in Chapter VII is retained. The property numbers are on the left, while the hypothesis numbers are on the right.

	$TP\uparrow \wedge I_{Gov.}\uparrow \Rightarrow FO\uparrow$	141
	$TP\uparrow \wedge \underline{CO}\uparrow \Rightarrow RG\uparrow$	138
	$TP\uparrow \wedge \sim\underline{CO}\uparrow \Rightarrow EF\downarrow$	139
	$TP\uparrow \wedge \overrightarrow{SZ} \Rightarrow FB\uparrow$	158
	$TP\uparrow \wedge FP\uparrow \wedge \overrightarrow{SZ} \Rightarrow FO\uparrow$	160
	$\overline{TP} \wedge FI\uparrow \wedge \overline{SB} \Rightarrow SB\uparrow$	162
	$\underline{TP} \wedge FI\downarrow \wedge \overline{SB} \Rightarrow SB\uparrow$	163
23. IP:	$IP\downarrow \Rightarrow FP\downarrow$	10
	$IP\downarrow \Rightarrow SP\downarrow$	11
	$IP\uparrow \Rightarrow FL\downarrow$	12
	$IP\downarrow \Rightarrow FL\uparrow$	13
	$\underline{IP} \Rightarrow \underline{RG}$	14
	$\overrightarrow{IP} \wedge \overrightarrow{FP} \Rightarrow \overrightarrow{OP}$	35
	$IP\uparrow \wedge \overrightarrow{SP} \Rightarrow FO\uparrow$	36
	$IP\uparrow \wedge \underline{SP} \Rightarrow SG_{Ref.}$	99
	$IP\uparrow \wedge \overline{SP} \Rightarrow \Delta IP = \Delta SP$	37
	$\Delta IP > \Delta FT \Rightarrow SL\uparrow$	38
	$\underline{IP} \wedge \overline{SL} \Rightarrow SP\uparrow$	39
	$\overline{IP} \wedge \overline{SL} \Rightarrow SP\downarrow$	40
	$\overrightarrow{IP} \wedge \overline{EF(t_1)} \Rightarrow EF\uparrow$	41
	$\frac{\max SI}{IP}\downarrow \Rightarrow FO\downarrow$	42
	$\Delta IP > \Delta FP$	50

24. FP: $FP\uparrow \wedge EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FO}$ 30[2]

$FP\uparrow \wedge EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FT}$ 31

$FP\uparrow \wedge \overline{\overline{TP}} \Rightarrow DC\uparrow$ 96

$FP\uparrow \wedge TP\uparrow \Rightarrow FT\uparrow$ 33

$F\vec{P} \wedge I\vec{P} \Rightarrow O\vec{P}$ 35

$FP\uparrow \wedge \overline{OP} \Rightarrow FO\downarrow$ 43

$F\vec{P} \vee FP\downarrow .\wedge\ CC\uparrow \wedge SR\uparrow .\Rightarrow FT\downarrow$ 140

$FP\uparrow \wedge TP\uparrow \wedge S\vec{Z} \Rightarrow FO\uparrow$ 160

$\overline{\Delta FP} \wedge \Delta SP < 0 < \Delta FP \wedge -\underline{\Delta SP} \Rightarrow EF\downarrow$ 44

$\Delta FP < \Delta IP$ 50

25. OP: $OP\uparrow \Rightarrow FP\uparrow$ 15

$\overline{OP} \wedge FP\uparrow \Rightarrow FO\downarrow$ 43

$OP\uparrow \wedge \underline{FB} \Rightarrow IP\uparrow$ 45

$O\vec{P} \wedge AM\downarrow \wedge \underline{IH} \Rightarrow FO\downarrow$ 161

26. SP: $SP\downarrow \Rightarrow FO\downarrow$ 16

$SP\uparrow \Rightarrow AD\uparrow$ 17

$SP\uparrow \Rightarrow EF\downarrow$ 18

$S\vec{P} \wedge IP\uparrow \Rightarrow FO\uparrow$ 36

$\underline{SP} \wedge IP\uparrow \Rightarrow SG_{Ref.}$ 99

$\overline{SP} \wedge IP\uparrow \Rightarrow \Delta IP = \Delta SP$ 37

$\Delta SP < 0 < \Delta FP \wedge \overline{\Delta FP} \wedge -\underline{\Delta SP} \Rightarrow EF\downarrow$ 44

$SP\uparrow \wedge .\ FL\downarrow \vee SL\downarrow .\Rightarrow TG\uparrow$ 46

$SP\uparrow \wedge .\ FL\downarrow \vee SL\downarrow .\Rightarrow IG\uparrow$ 98

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[2] Hypotheses may occur more than once in the listing, since hypotheses may be reordered. To illustrate, hypothesis 30 occurs under the property $EC_{\overline{S}}$ when $EC_{\overline{S}}\uparrow$ is the first conjunct but under the property FP when FP↑ is the first conjunct.

27. FI: $FI\uparrow \Rightarrow FP\curvearrowright$ 19

$FI\uparrow \Rightarrow SL\uparrow$ 20

$FI\downarrow \Rightarrow U\downarrow$ 91

$\overline{FI}\downarrow \Rightarrow HO\downarrow$ 92

$FI\downarrow \Rightarrow XD\uparrow$ 93

$\overrightarrow{FI} \Rightarrow \overline{HS}$ 143

$FI \equiv FO \Leftrightarrow EF = \max EF$ 52

$FI\uparrow \wedge \overline{\overline{CP}} \Rightarrow DC\uparrow$ 97

$FI\uparrow \wedge \overline{TP} \wedge \overline{SB} \Rightarrow SB\uparrow$ 162

$FI\downarrow \wedge \underline{TP} \wedge \overline{SB} \Rightarrow SB\uparrow$ 163

$FI\uparrow \wedge \overrightarrow{FO} \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge FO\uparrow \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge \overrightarrow{FO} \wedge CP\downarrow\ .\Rightarrow O\uparrow$ 48

$FI\downarrow \wedge \overrightarrow{FO} \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge FO\downarrow \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge \overrightarrow{FO} \wedge CP\uparrow\ .\Rightarrow O\downarrow$ 49

$\Delta FI > \Delta FO$ 51

28. FO: $\overline{FO} \Rightarrow XD\uparrow$ 94

$\overline{\overline{FO}} \wedge TP\uparrow \Rightarrow SE\uparrow$ 147

$FO \equiv FI \Leftrightarrow EF = \max EF$ 52

$\underline{FO} \wedge \overline{CO} \Rightarrow \overline{SG}$ 137

$\overrightarrow{FO} \wedge FI\uparrow \wedge \overrightarrow{CP}\ .\vee.\ FO\uparrow \wedge \overrightarrow{FI} \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FO} \wedge \overrightarrow{FI} \wedge CP\downarrow\ .\Rightarrow O\uparrow$ 48

$\overrightarrow{FO} \wedge FI\downarrow \wedge \overrightarrow{CP}\ .\vee.\ FO\downarrow \wedge \overrightarrow{FI} \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FO} \wedge \overrightarrow{FI} \wedge CP\uparrow\ .\Rightarrow O\downarrow$ 49

$\Delta FO < \Delta FI$ 51

29. FT: $FT\uparrow \Rightarrow CP\uparrow$ 21

$FT\uparrow \Rightarrow \overline{WE}$ 95

$\overline{FT} \Rightarrow \underline{FL} \vee \underline{SL}$ 22

$\underline{FT} \wedge \underline{EC_{\overline{S}}} \Rightarrow \underline{SB}$ 32

$\Delta FT < \Delta IP \Rightarrow SL\uparrow$ 38

$\sim\underline{FT} \wedge \underline{EC_{\overline{S}}} \wedge \underline{FB} \Rightarrow \overline{SB}$ 148

$\underline{FT} \wedge \underline{FB} \wedge \overline{SL} \Rightarrow \underline{EF}$ 47

30. FB: $\underline{\Delta FB} \Rightarrow EC_{\overline{S}}\uparrow$ 23

$\underline{FB} \Rightarrow \overline{SP}$ 24

$\underline{FB} \Rightarrow \overline{RG}$ 25

$\underline{FB} \wedge OP\uparrow \Rightarrow IP\uparrow$ 45

$\sim\underline{FT} \wedge \underline{EC_{\overline{S}}} \wedge \underline{FB} \Rightarrow \overline{SB}$ 148

$\underline{FB} \wedge \underline{FT} \wedge \overline{SL} \Rightarrow \underline{EF}$ 47

$\underline{FB} \wedge D_{P\ Pun.} \wedge \underline{D_A} \Rightarrow \underline{EF}$ 142

31. FL: $\overline{FL} \Rightarrow \overline{CP}$ 27

$\underline{FL} \Rightarrow \underline{CP}$ 26

$FL\uparrow \Rightarrow AD\uparrow$ 28

$FL\downarrow \Rightarrow IM\uparrow$ 144

$FL_{Ins.}\uparrow \Rightarrow IM_{Ins.}\uparrow$ 149

$\underline{FL} \Rightarrow \underline{SB}$ 145

$FL\downarrow \vee SL\downarrow .\wedge SP\uparrow. \Rightarrow TG\uparrow$ 46

$FL\downarrow \vee SL\downarrow .\wedge SP\uparrow. \Rightarrow IG\uparrow$ 98

32. SL: $\overline{SL} \wedge \underline{IP} \Rightarrow SP\uparrow$ 39

$\overline{SL} \wedge \overline{IP} \Rightarrow SP\downarrow$ 40

$SL\downarrow \vee FL\downarrow .\wedge SP\uparrow. \Rightarrow TG\uparrow$ 46

$SL\downarrow \vee FL\downarrow .\wedge SP\uparrow. \Rightarrow IG\uparrow$ 98

$\overline{SL} \wedge \underline{FT} \wedge \underline{FB} \Rightarrow \underline{EF}$ 47

33. RG:

34. CP: $\overline{\overline{CP}} \wedge FI\uparrow \Rightarrow DC\uparrow$ 97

$\underline{CP} \wedge \underline{EC_{\overline{S}}} \wedge \underline{SB} \Rightarrow \underline{SP} \vee \underline{FL} \vee \underline{SL}$ 159

$\vec{CP} \wedge FI\uparrow \wedge \vec{FO} \;.\vee.\; \vec{CP} \wedge \vec{FI} \wedge FO\uparrow \;.\vee.\; CP\downarrow \wedge \vec{FI} \wedge \vec{FO} \;.\Rightarrow O\uparrow$ 48

$\vec{CP} \wedge FI\downarrow \wedge \vec{FO} \;.\vee.\; \vec{CP} \wedge \vec{FI} \wedge FO\downarrow \;.\vee.\; CP\uparrow \wedge \vec{FI} \wedge \vec{FO} \;.\Rightarrow O\downarrow$ 49

35. O: $O\uparrow \Rightarrow EF\downarrow$ 29

36. AD: $\underline{AD} \Rightarrow SB\downarrow$ 146

37. EF: $\underline{EF} \wedge \vec{TP} \Rightarrow \overline{RG}$ 34

$\overline{EF(t_1)} \wedge \vec{IP} \Rightarrow EF\uparrow$ 41

$EF = \max EF \Leftrightarrow FI \equiv FO$ 52

$\underline{EF} \wedge \underline{CO} \Rightarrow \underline{SD}$ 191

38. CC: $CC\uparrow \Rightarrow FI\uparrow$ 100

$CC\uparrow \Rightarrow F\uparrow$ 53

$CC\uparrow \vee SR\uparrow \Rightarrow TP\uparrow$ 106

$CC\uparrow \vee SR\uparrow \Rightarrow IP\uparrow$ 107

$CC\uparrow \vee SR\uparrow \Rightarrow FL\downarrow$ 108

$CC\uparrow \vee SR\uparrow \Rightarrow SL\uparrow$ 109

$CC\uparrow \vee SR\uparrow \Rightarrow 0 < \Delta FP < \Delta IP$ 110

$CC\uparrow \vee SR\uparrow \Rightarrow \Delta FP < \Delta SP$ 111

$CC\uparrow \wedge SR\uparrow \;\wedge.\; \vec{FP} \vee FP\downarrow \;.\Rightarrow FT\downarrow$ 140

$CC_{\text{Ins.-Ref.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Ref.}} \wedge W_{\text{Ins.-Ref.}}\uparrow \Rightarrow IP\uparrow \wedge FP\uparrow \wedge FO\downarrow \wedge RG\uparrow$ 130

$CC_{\text{Ins.-Exp.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Exp.}} \wedge W_{\text{Ins.-Exp.}}\uparrow \Rightarrow IP\uparrow \wedge SP\uparrow \wedge FO\uparrow \wedge FL\uparrow$ 131

39. SR: $SR_{Gov.} \Rightarrow CC_{Ref.}$ 73

$SR_{Ref.} \Rightarrow VN_{Gov.}\downarrow$ 74

$SR_{Ref.} \Rightarrow VN_{Ref.}\downarrow$ 75

$SR\downarrow \Rightarrow W\uparrow$ 54

$SR\uparrow \Rightarrow HO\downarrow$ 55

$SR\uparrow \Rightarrow F\uparrow$ 56

$\underline{SR}_{Rew.} \Rightarrow CC_{Ref.}\uparrow \vee SR_{Ref.}\uparrow$ 76

$\underline{SR}_{Rew.} \Rightarrow W_{Gov.} \wedge HO_{Gov.}$ 77

$SR\uparrow \vee CC\uparrow \Rightarrow TP\uparrow$ 106

$SR\uparrow \vee CC\uparrow \Rightarrow IP\uparrow$ 107

$SR\uparrow \vee CC\uparrow \Rightarrow FL\downarrow$ 108

$SR\uparrow \vee CC\uparrow \Rightarrow SL\uparrow$ 109

$SR\uparrow \vee CC\uparrow \Rightarrow 0 < \Delta FP < \Delta IP$ 110

$SR\uparrow \vee CC\uparrow \Rightarrow \Delta FP < \Delta SP$ 111

$SR\uparrow \wedge H\vec{O} \Rightarrow RG\downarrow$ 112

$SR_{Gov.}\uparrow \wedge HO_{Gov.}\downarrow \Rightarrow SR_{Ref.}\uparrow$ 78

$\underline{SR}_{Ref.} \wedge \underline{HO}_{Gov.} \Rightarrow W_{Gov.}$ 79

$\overline{SR}_{Ref.} \wedge CE_{Gov.} \Rightarrow W_{Gov.}$ 80

$SR\uparrow \wedge CC\uparrow \;\wedge.\; F\vec{P} \vee FP\downarrow \;.\!\Rightarrow FT\downarrow$ 140

$\overline{SR}_{Ref.} \wedge CE_{Gov.} \wedge \underline{HO}_{Gov.} \Rightarrow CO_{Gov.}\uparrow$ 81

40. U: $U \Rightarrow HO$ 57

$U \vee WE\uparrow \vee DC\uparrow \Rightarrow IP\downarrow \wedge FP\downarrow$ 115

$\lim_{U\uparrow} \frac{D_A}{D_P} = 1$ 71

41. WE: $\underline{WE} \Rightarrow \overline{FT}$ 101

$WE \wedge HO \Rightarrow F\downarrow$ 114

$\max WE \wedge SZ\uparrow \Rightarrow D_P\uparrow \vee D_A\uparrow$ 177

$WE\uparrow \vee U \vee DC\uparrow \Rightarrow IP\downarrow \wedge FP\downarrow$ 115

42. DC: $\underline{DC} \Rightarrow I\uparrow$ 58

$\underline{DC} \Rightarrow SG\uparrow$ 59

$\underline{DC}_{\mathrm{Fac.}} \wedge \overline{W}_{\mathrm{Fac.}} \Rightarrow \overline{SD}_{\mathrm{Fac.}}$ 171

$DC\uparrow \vee U \vee WE\uparrow \Rightarrow IP\downarrow \wedge FP\downarrow$ 115

$\underline{DC}_{\mathrm{Ins.-Ref.}} \wedge CC_{\mathrm{Ins.-Ref.}}\uparrow \wedge W_{\mathrm{Ins.-Ref.}}\uparrow \Rightarrow IP\uparrow \wedge FP\uparrow \wedge FO\downarrow \wedge RG\uparrow$ 130

$\underline{DC}_{\mathrm{Ins.-Exp.}} \wedge CC_{\mathrm{Ins.-Exp.}}\uparrow \wedge W_{\mathrm{Ins.-Exp.}}\uparrow \Rightarrow IP\uparrow \wedge SP\uparrow \wedge FO\uparrow \wedge FL\uparrow$ 131

$\underline{DC}_{\mathrm{Ins.-Ref.}} \wedge D_P\ \mathrm{Ins.-Ref.}\uparrow \wedge W_{\mathrm{Ins.-Ref.}}\uparrow \Rightarrow IP\downarrow \wedge FP\downarrow \wedge FO\downarrow \wedge RG\downarrow$ 132

$\underline{DC}_{\mathrm{Ins.-Leg.}} \wedge D_P\ \mathrm{Ins.-Leg.}\uparrow \wedge W_{\mathrm{Ins.-Leg.}}\uparrow \Rightarrow FO\uparrow \wedge \underline{SL} \wedge \underline{RG}$ 134

$\underline{DC}_{\mathrm{Ins.-Rew.}} \wedge D_P\ \mathrm{Ins.-Rew.}\uparrow \wedge W_{\mathrm{Ins.-Rew.}}\uparrow \Rightarrow.$

$\underline{EC_{\overline{S}}} \Rightarrow \underline{AD} \ .\wedge\ \overline{IP} \wedge \overline{SP} \wedge \underline{FL}$ 133

$\underline{DC}_{\mathrm{Ins.-Pun.}} \wedge D_P\ \mathrm{Ins.-Pun.}\uparrow \wedge W_{\mathrm{Ins.-Pun.}}\uparrow \wedge HO_{\mathrm{Ins.-Pun.}}\uparrow \Rightarrow.$

$\underline{EC_{\overline{S}}} \Rightarrow \overline{AD} \ .\wedge\ FP\downarrow \wedge FO\downarrow \wedge RG\downarrow \wedge SB\uparrow \wedge EL\uparrow$ 135

$\underline{DC}_{\mathrm{Ins.}}$ 84

$\underline{DC}_{\mathrm{Inq.}}$ 85

43. V: $VN\uparrow \Rightarrow CC\downarrow$ 60

44. D_P: $D_{P\ \text{Rew.}}\uparrow \Rightarrow FO\downarrow$ 116

$D_{P\ \text{Rew.}}\uparrow \Rightarrow \underline{AD}\uparrow$ 117

$D_P\uparrow \Rightarrow CE\uparrow$ 61

$\max D_{P\ \text{Gov.-Leg.}} \Rightarrow W\uparrow \wedge HO\uparrow \wedge CE\uparrow$ 72

$D_{P\ \text{Inq.-Leg.}}\uparrow \Rightarrow FO\uparrow \wedge SL\uparrow \wedge \underline{\max SI}$ 126

$D_{P\ \text{Inq.-Exp.}}\uparrow \Rightarrow FO\downarrow \wedge \underline{SL}\uparrow \wedge \overline{\max SI}$ 127

$D_{P\ \text{Pun.}} \wedge FB \wedge \underline{D_A} \Rightarrow \underline{EF}$ 142

$D_{P\ \text{Ins.-Ref.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Ref.}} \wedge W_{\text{Ins.-Ref.}}\uparrow \Rightarrow$

$IP\downarrow \wedge FP\downarrow \wedge FO\downarrow \wedge RG\downarrow$ 132

$D_{P\ \text{Ins.-Leg.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Leg.}} \wedge W_{\text{Ins.-Leg.}}\uparrow \Rightarrow FO\uparrow \wedge \underline{SL} \wedge \underline{RG}$ 134

$D_{P\ \text{Ins.-Rew.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Rew.}} \wedge W_{\text{Ins.-Rew.}}\uparrow \Rightarrow.$

$\underline{EC}_{\overline{S}} \Rightarrow \underline{AD}\ .\wedge\ \overline{IP} \wedge \overline{SP} \wedge \underline{FL}$ 133

$D_{P\ \text{Ins.-Pun.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Pun.}} \wedge W_{\text{Ins.-Pun.}}\uparrow \wedge HO_{\text{Ins.-Pun.}}\uparrow \Rightarrow.$

$\underline{EC}_{\overline{S}} \Rightarrow \overline{AD}\ .\wedge\ FP\downarrow \wedge FO\downarrow \wedge RG\downarrow \wedge SB\uparrow \wedge EL\uparrow$ 135

$\lim\limits_{U\uparrow} \frac{D_A}{D_P} = 1$ 71

45. D_A: $\underline{D_A}_{\ \text{Fac.-Leg.}} \Rightarrow \overline{RG}$ 128

$D_A\uparrow \Rightarrow CE\downarrow$ 62

$\max D_{A\ \text{Inq.}_{\text{Res.}}\text{-Leg.}} \Rightarrow FP\uparrow \wedge IP\uparrow \wedge SP\uparrow \wedge FL\uparrow \wedge AM_{\text{Ins.}}\uparrow$ 189

$\max D_{A\ \text{Inq.}_{\text{Dev.}}\text{-Leg.}} \Rightarrow \overline{FP} \wedge FL\uparrow \wedge SL\uparrow \wedge \overline{RG} \wedge D_{A\ \text{Ins.}}\uparrow \wedge D_{A\ \text{Inq.}}\downarrow$ 136

$\underline{D_A} \wedge D_{P\ \text{Pun.}} \wedge \underline{FB} \Rightarrow \underline{EF}$ 142

$\lim_{U\uparrow} \frac{D_A}{D_P} = 1$ 71

46. I: $I_{\text{Gov.}}\uparrow \Rightarrow FP\uparrow$ 118

$I_{\text{Gov.}}\uparrow \Rightarrow \overline{OP}$ 119

$I_{\text{Gov.}}\uparrow \Rightarrow FO\downarrow$ 120

$I\uparrow \Rightarrow \overline{SB}$ 164

$I_{\text{Gov.}}\uparrow \wedge TP\uparrow \Rightarrow FO\uparrow$ 141

$I\uparrow \wedge W\uparrow \Rightarrow \underline{SS}$ 168

47. SG:

48. ID: $ID\uparrow \Rightarrow FI\uparrow$ 102

$ID\uparrow \Rightarrow XG\uparrow$ 63

49. W: $W\uparrow \Rightarrow \overline{RG}$ 103

$\underline{W}_{\text{Ref.}} \Rightarrow \overline{\overline{O}}$ 122

$\underline{W}_{\text{Ref.}} \Rightarrow \underline{|\max FP - FP|}$ 121

$W_{\text{Ref.}} \Rightarrow CC_{\text{Ref.}}\uparrow \vee SR_{\text{Ref.}}\uparrow$ 82

$W_{\text{Inq.-Ref.}}\uparrow \Rightarrow \frac{\max SI}{IP}\uparrow$ 129

$\overline{W}_{\text{Fac.}} \wedge \underline{DC}_{\text{Fac.}} \Rightarrow \overline{SD}_{\text{Fac.}}$ 171

$W\uparrow \wedge I\uparrow \Rightarrow \underline{SS}$ 168

$W\uparrow \wedge \overrightarrow{HO} \Rightarrow EF\downarrow$ 113

$W\uparrow \wedge \overrightarrow{HO} \Rightarrow IG\uparrow$ 70

$\underline{W} \wedge \underline{CE} \Rightarrow \underline{SD}$ 169

$W_{\text{Ins.-Ref.}}\uparrow \wedge CC_{\text{Ins.-Ref.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Ref.}} \Rightarrow$

$IP\uparrow \wedge FP\uparrow \wedge FO\downarrow \wedge RG\uparrow$ 130

$W_{\text{Ins.-Exp.}}\uparrow \wedge CC_{\text{Ins.-Exp.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Exp.}} \Rightarrow$

$IP\uparrow \wedge SP\uparrow \wedge FO\uparrow \wedge FL\uparrow$ 131

$W_{\text{Ins.-Ref.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Ref.}} \wedge D_{P\ \text{Ins.-Ref.}}\uparrow \Rightarrow$

$IP\downarrow \wedge FP\downarrow \wedge FO\downarrow \wedge RG\downarrow$ 132

$W_{\text{Ins.-Leg.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Leg.}} \wedge D_{P\ \text{Ins.-Leg.}}\uparrow \Rightarrow FO\uparrow \wedge \underline{SL} \wedge \underline{RG}$ 134

$W_{\text{Ins.-Rew.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Rew.}} \wedge D_{P\ \text{Ins.-Rew.}}\uparrow \Rightarrow.$

$\underline{EC}_{\overline{S}} \Rightarrow \underline{AD}\ .\wedge\ \overline{IP} \wedge \overline{SP} \wedge \underline{FL}$ 133

$W_{\text{Ins.-Pun.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Pun.}} \wedge D_{P\ \text{Ins.-Pun.}}\uparrow \wedge HO_{\text{Ins.-Pun.}}\uparrow \Rightarrow.$

$\underline{EC}_{\overline{S}} \Rightarrow \overline{AD}\ .\wedge\ FP\downarrow \wedge FO\downarrow \wedge RG\downarrow \wedge SB\uparrow \wedge EL\uparrow$ 135

50. IG:

51. HO: $HO_{\text{Gov.}}\uparrow \Rightarrow FL\uparrow$ 123

$HO\uparrow \Rightarrow VN\uparrow \wedge F\downarrow$ 64

$\overrightarrow{HO} \wedge SR\uparrow \Rightarrow RG\downarrow$ 112

$HO_{\text{Gov.}}\downarrow \wedge SR_{\text{Gov.}}\uparrow \Rightarrow SR_{\text{Ref.}}\uparrow$ 78

$\underline{HO}_{\text{Gov.}} \wedge \underline{SR}_{\text{Ref.}} \Rightarrow W_{\text{Gov.}}$ 79

$HO_{\text{Ins.-Pun.}}\uparrow \wedge \underline{DC}_{\text{Ins.-Pun.}} \wedge D_{P\ \text{Ins.-Pun.}}\uparrow \wedge W_{\text{Ins.-Pun.}}\uparrow \Rightarrow.$

$\underline{EC}_{\overline{S}} \Rightarrow \overline{AD}\ .\wedge\ FP\downarrow \wedge FO\downarrow \wedge RG\downarrow \wedge SB\uparrow \wedge EL\uparrow$ 135

$HO \wedge WE \Rightarrow F\downarrow$ 114

$\vec{HO} \wedge W\uparrow \Rightarrow EF\downarrow$ 113

$\vec{HO} \wedge W\uparrow \Rightarrow IG\uparrow$ 70

$\underline{HO}_{Gov.} \wedge \underline{F}_{Gov.} \Rightarrow DC_{Ref.}$ 83

$\underline{HO}(t_1) \wedge \underline{SZ}(t_1) \Rightarrow I(t_2)\uparrow$ 178

$\underline{HO}_{Gov.} \wedge \overline{SR}_{Ref.} \wedge CE_{Gov.} \Rightarrow CO_{Gov.}\uparrow$ 81

$\Delta HO < \Delta SZ$ 174

52. F: $F\downarrow \Rightarrow SD\uparrow$ 165

$\underline{F}_{Gov.} \wedge \underline{HO}_{Gov.} \Rightarrow DC_{Ref.}$ 83

53. HM: $HM(t_2) > HM(t_1) \Rightarrow \underline{\underline{TP}} \wedge \underline{\underline{ZD}} \wedge \underline{\underline{XD}}$ 190

$\underline{HM} \wedge \vec{OP} \wedge AM\downarrow \Rightarrow FO\downarrow$ 161

54. IM: $IM\uparrow \Rightarrow FP\downarrow \wedge FO\downarrow$ 151

$IM_{Ins.}\uparrow \Rightarrow FP\downarrow \wedge FO\downarrow$ 157

55. AM: $AM\uparrow \Rightarrow W\downarrow$ 172

$AM\uparrow \Rightarrow CE\downarrow$ 173

$AM\uparrow \Rightarrow IP\uparrow \wedge FP\downarrow \wedge SP\uparrow \wedge FO\downarrow \wedge FL\downarrow \wedge SL\downarrow \wedge EF\downarrow$ 150

$AM\downarrow \wedge \vec{OP} \wedge \underline{HM} \Rightarrow FO\downarrow$ 161

56. CO: $\underline{CO}\uparrow \Rightarrow EF\uparrow$ 104

$CO\uparrow \Rightarrow HO\downarrow$ 65

$\underline{CO}\uparrow \wedge TP\uparrow \Rightarrow RG\uparrow$ 138

$\sim\underline{CO}\uparrow \wedge TP\uparrow \Rightarrow EF\downarrow$ 139

$\overline{CO} \wedge \underline{FO} \Rightarrow \overline{SG}$ 137

$\underline{CO} \wedge \underline{EF} \Rightarrow \underline{SD}$ 191

57. CE:	$CE\uparrow \Rightarrow TP\downarrow$	105
	$CE\uparrow \Rightarrow D_P\uparrow$	66
	$CE\uparrow \Rightarrow D_A\downarrow$	67
	$\overline{CE} \Rightarrow I\uparrow$	68
	$CE_{Ins.}\uparrow \Rightarrow IM_{Ins.}\uparrow$	170
	$\overline{CE} \Rightarrow CE\uparrow$	69
	$CE\uparrow \Rightarrow SS\uparrow$	166
	$CE_{Gov.} \wedge \overline{SR}_{Ref.} \Rightarrow W_{Gov.}$	80
	$\underline{CE} \wedge \underline{W} \Rightarrow \underline{SD}$	169
	$CE\uparrow \wedge \underline{SE} \Rightarrow SB\downarrow$	186
	$CE\uparrow \wedge SE = 0 \Rightarrow SB\uparrow$	187
	$CE_{Gov.} \wedge \overline{SR}_{Ref.} \wedge \underline{HO}_{Gov.} \Rightarrow CO_{Gov.}\uparrow$	81
58. SZ:	$SZ\uparrow \wedge \max WE \Rightarrow D_P\uparrow \vee D_A\uparrow$	177
	$\underline{SZ}(t_1) \wedge \underline{HO}(t_1) \Rightarrow I(t_2)\uparrow$	178
	$S\vec{Z} \wedge TP\uparrow \Rightarrow FB\uparrow$	158
	$S\vec{Z} \wedge TP\uparrow \wedge FP\uparrow \Rightarrow FO\uparrow$	160
	$SZ\uparrow \wedge X\vec{G} \Rightarrow TP\uparrow$	194
	$SZ\uparrow \wedge X\vec{G} \Rightarrow F!\downarrow$	195
	$SZ\uparrow \wedge X\vec{G} \Rightarrow FO\uparrow \wedge \Delta FO\downarrow$	196
	$SZ\uparrow \wedge X\vec{G} \Rightarrow FT\uparrow$	197
	$SZ\uparrow \wedge X\vec{G} \Rightarrow FB\downarrow$	198
	$SZ\uparrow \wedge X\vec{G} \Rightarrow RG\curvearrowright$	199

	$SZ\uparrow \wedge X\vec{G} \Rightarrow CP\downarrow$	200
	$SZ\uparrow \wedge X\vec{G} \Rightarrow EF\curvearrowright$	201
	$SZ\uparrow \wedge X\vec{G} \Rightarrow VN\uparrow$	179
	$SZ\uparrow \wedge X\vec{G} \Rightarrow F\downarrow$	180
	$SZ\uparrow \wedge X\vec{G} \Rightarrow CE\downarrow$	181
	$SZ\uparrow \wedge X\vec{G} \Rightarrow SD\uparrow$	188
	$S\vec{Z} \wedge XD\uparrow \Rightarrow DC\uparrow$	182
	$SZ\downarrow \wedge XD\uparrow \Rightarrow DC\downarrow$	183
	$\Delta SZ > \Delta HO$	174
59. CX:	$\underline{CX}_{Fac.} \Rightarrow \overline{FT}_{Fac.}$	125
	$\underline{CX}_{Fac.} \Rightarrow \underline{RG}$	124
	$\underline{CX}\uparrow \Rightarrow SZ\uparrow$	167
	$CX\uparrow \wedge Z\vec{G} \Rightarrow CO\downarrow$	184
	$CX\uparrow \wedge Z\vec{G} \Rightarrow CE\uparrow$	185
60. SI:	$\frac{\max SI}{IP}\downarrow \Rightarrow FO\downarrow$	42
61. ZG:	$Z\vec{G} \wedge CX\uparrow \Rightarrow CO\downarrow$	184
	$Z\vec{G} \wedge CX\uparrow \Rightarrow CE\uparrow$	185
	$ZG\downarrow \wedge T\vec{G} \Rightarrow XG\uparrow$	192

62. XG: $X\vec{G} \wedge SZ\uparrow \Rightarrow TP\uparrow$ 194
$X\vec{G} \wedge SZ\uparrow \Rightarrow FI\downarrow$ 195
$X\vec{G} \wedge SZ\uparrow \Rightarrow FO\uparrow \wedge \Delta FO\downarrow$ 196
$X\vec{G} \wedge SZ\uparrow \Rightarrow FT\uparrow$ 197
$X\vec{G} \wedge SZ\uparrow \Rightarrow FB\downarrow$ 198
$X\vec{G} \wedge SZ\uparrow \Rightarrow RG\curvearrowright$ 199
$X\vec{G} \wedge SZ\uparrow \Rightarrow CP\downarrow$ 200
$X\vec{G} \wedge SZ\uparrow \Rightarrow EF\curvearrowright$ 201
$X\vec{G} \wedge SZ\uparrow \Rightarrow VN\uparrow$ 179
$X\vec{G} \wedge SZ\uparrow \Rightarrow F\downarrow$ 180
$X\vec{G} \wedge SZ\uparrow \Rightarrow CE\downarrow$ 181
$X\vec{G} \wedge SZ\uparrow \Rightarrow SD\uparrow$ 188

63. TG: $T\vec{G} \wedge ZG\downarrow \Rightarrow XG\uparrow$ 192

64. ZD: $ZD\downarrow \wedge T\vec{D} \Rightarrow XD\uparrow$ 193

65. XD: $XD\uparrow \Rightarrow ZD\uparrow \vee DC\uparrow$ 175
$XD\uparrow \wedge S\vec{Z} \Rightarrow DC\uparrow$ 182
$XD\uparrow \wedge SZ\downarrow \Rightarrow DC\downarrow$ 183

66. TD: $T\vec{D} \wedge ZD\downarrow \Rightarrow XD\uparrow$ 193

67. SB: $\underline{SB} \wedge \underline{EC}_{S} \wedge \underline{CP} \Rightarrow \underline{SP} \vee \underline{FL} \vee \underline{SL}$ 159
$\overline{SB} \wedge \overline{TP} \wedge FI\uparrow \Rightarrow SB\uparrow$ 162
$\overline{SB} \wedge \underline{TP} \wedge FI\downarrow \Rightarrow SB\uparrow$ 163

68. SS: $\underline{SS} \Rightarrow \overline{AD}$ 152

$\underline{SS} \Rightarrow SA\uparrow$ 86

$\overline{SS} \Rightarrow \overline{SG} \wedge \overline{IG} \wedge \overline{HS}$ 176

$SS\uparrow \Leftrightarrow SD\uparrow \ .\wedge.\ SS\downarrow \Leftrightarrow SD\downarrow$ 89

69. SD: $SD\uparrow \Rightarrow RG\downarrow$ 153

$SD\uparrow \Rightarrow SI\downarrow$ 154

$SD\uparrow \Leftrightarrow SS\uparrow \ .\wedge.\ SD\downarrow \Leftrightarrow SS\downarrow$ 89

70. EL: $\underline{EL} \Rightarrow \overline{RG}$ 155

$EL(t_1) \wedge \underline{HS} \Rightarrow \overline{RG}$ 156

71. HS: $\underline{HS} \wedge EL(t_1) \Rightarrow \overline{RG}$ 156

72. SE: $\overline{SE} \Rightarrow S\vec{S}$ 87

$\underline{SE}\uparrow \Rightarrow SA\uparrow$ 88

$\underline{SE} \wedge CE\uparrow \Rightarrow SB\downarrow$ 186

$SE = 0 \wedge CE\uparrow \Rightarrow SB\uparrow$ 187

73. SA:

Appendix IV

List of Hypotheses According to the Interrelations of Set Theory, Information Theory, and Graph Theory

I. Information Theoretic Hypotheses

1. $EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta IP}$
2. $EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FP}$
3. $EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FB}$
4. $EC_{\overline{S}}\uparrow \Rightarrow \underline{\Delta FL}$
5. $TP\uparrow \Rightarrow IP\curvearrowright$
6. $\underline{TP}\uparrow \Rightarrow FP\uparrow$
7. $\overline{\overline{TP}} \Rightarrow FP\uparrow$
8. $TP\uparrow \Rightarrow FL\circlearrowleft$
9. $TP\uparrow \Rightarrow \overline{RG}\uparrow$
10. $IP\downarrow \Rightarrow FP\downarrow$
11. $IP\downarrow \Rightarrow SP\downarrow$
12. $IP\uparrow \Rightarrow FL\downarrow$
13. $IP\downarrow \Rightarrow FL\uparrow$
14. $\underline{IP} \Rightarrow \underline{RG}$
15. $OP\uparrow \Rightarrow FP\uparrow$
16. $SP\downarrow \Rightarrow FO\downarrow$
17. $SP\uparrow \Rightarrow AD\uparrow$
18. $SP\uparrow \Rightarrow EF\downarrow$

19. $FI\uparrow \Rightarrow FP\Omega$

20. $FI\uparrow \Rightarrow SL\uparrow$

21. $FT\uparrow \Rightarrow CP\uparrow$

22. $\overline{FT} \Rightarrow \underline{FL} \vee \underline{SL}$

23. $\underline{\Delta FB} \Rightarrow EC_{\overline{S}}\uparrow$

24. $\underline{FB} \Rightarrow \overline{SP}$

25. $\underline{FB} \Rightarrow \overline{RG}$

26. $\underline{FL} \Rightarrow \underline{CP}$

27. $\overline{FL} \Rightarrow \overline{CP}$

28. $FL\uparrow \Rightarrow AD\uparrow$

29. $O\uparrow \Rightarrow EF\downarrow$

30. $EC_{\overline{S}}\uparrow \wedge FP\uparrow \Rightarrow \underline{\Delta FO}$

31. $EC_{\overline{S}}\uparrow \wedge FP\uparrow \Rightarrow \underline{\Delta FT}$

32. $\underline{EC_{\overline{S}}} \wedge \underline{FT} \Rightarrow \underline{SB}$

33. $TP\uparrow \wedge FP\uparrow \Rightarrow FT\uparrow$

34. $T\overset{\rightrightarrows}{P} \wedge \underline{EF} \Rightarrow \overline{RG}$

35. $I\overset{\rightrightarrows}{P} \wedge F\overset{\rightrightarrows}{P} \Rightarrow O\overset{\rightrightarrows}{P}$

36. $IP\uparrow \wedge S\overset{\rightrightarrows}{P} \Rightarrow FO\uparrow$

37. $IP\uparrow \wedge \overline{SP} \Rightarrow \Delta IP = \Delta SP$

38. $\Delta IP > \Delta FT \Rightarrow SL\uparrow$

39. $\underline{IP} \wedge \overline{SL} \Rightarrow SP\uparrow$

40. $\overline{IP} \wedge \overline{SL} \Rightarrow SP\downarrow$

41. $I\overset{\rightrightarrows}{P} \wedge \overline{EF(t_1)} \Rightarrow EF\uparrow$

42. $\frac{\max SI}{IP}\downarrow \Rightarrow FO\downarrow$

43. $FP\uparrow \wedge \overline{OP} \Rightarrow FO\downarrow$

44. $\overline{\Delta FP} \wedge \Delta SP < 0 < \Delta FP \wedge -\underline{\Delta SP} \Rightarrow EF\downarrow$

45. $OP\uparrow \wedge \underline{FB} \Rightarrow IP\uparrow$

46. $SP\uparrow \wedge .\ FL\downarrow \vee SL\downarrow\ .\Rightarrow TG\uparrow$

47. $\underline{FT} \wedge \underline{FB} \wedge \overline{SL} \Rightarrow \underline{EF}$

48. $FI\uparrow \wedge \overrightarrow{FO} \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge FO\uparrow \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge \overrightarrow{FO} \wedge CP\downarrow\ .\Rightarrow O\uparrow$

49. $FI\downarrow \wedge \overrightarrow{FO} \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge FO\downarrow \wedge \overrightarrow{CP}\ .\vee.\ \overrightarrow{FI} \wedge \overrightarrow{FO} \wedge CP\uparrow\ .\Rightarrow O\downarrow$

50. $\Delta IP > \Delta FP$

51. $\Delta FI > \Delta FO$

52. $EF = \max EF \Leftrightarrow FI \equiv FO$

2. Graph Theoretic Hypotheses

2.1. Graph Theoretic Antecedent--Graph Theoretic Consequent

53. $CC\uparrow \Rightarrow F\uparrow$

54. $SR\downarrow \Rightarrow W\uparrow$

55. $SR\uparrow \Rightarrow HO\downarrow$

56. $SR\uparrow \Rightarrow F\uparrow$

57. $U \Rightarrow HO$

58. $\underline{DC} \Rightarrow I\uparrow$

59. $\underline{DC} \Rightarrow SG\uparrow$

60. $VN\uparrow \Rightarrow CC\downarrow$

61. $D_P\uparrow \Rightarrow CE\uparrow$

62. $D_A\uparrow \Rightarrow CE\downarrow$

63. $ID\uparrow \Rightarrow XG\uparrow$

64. $HO\uparrow \Rightarrow VN\uparrow \wedge F\downarrow$

65. $CO\uparrow \Rightarrow HO\downarrow$

66. $CE\uparrow \Rightarrow D_P\uparrow$

67. $CE\uparrow \Rightarrow D_A\downarrow$

68. $\overline{CE} \Rightarrow I\uparrow$

69. $\overline{CE} \Rightarrow CE\uparrow$

70. $W\uparrow \wedge \overset{\Rightarrow}{HO} \Rightarrow IG\uparrow$

71. $\lim\limits_{U\uparrow} \frac{D_A}{D_P} = 1$

2.1.1. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Graph Theoretic Consequent

72. $\max D_{P\ \text{Gov.-Leg.}} \Rightarrow W\uparrow \wedge HO\uparrow \wedge CE\uparrow$

2.1.2. Both Graph Theoretic Antecedent and Consequent with Respect to Affect Relations

73. $SR_{\text{Gov.}} \Rightarrow CC_{\text{Ref.}}$

74. $SR_{\text{Ref.}} \Rightarrow VN_{\text{Gov.}}\downarrow$

75. $SR_{\text{Ref.}} \Rightarrow VN_{\text{Ref.}}\downarrow$

76. $\underline{SR}_{\text{Rew.}} \Rightarrow CC_{\text{Ref.}}\uparrow \vee SR_{\text{Ref.}}\uparrow$

77. $\underline{SR}_{\text{Rew.}} \Rightarrow W_{\text{Gov.}} \wedge HO_{\text{Gov.}}$

78. $SR_{\text{Gov.}}\uparrow \wedge HO_{\text{Gov.}}\downarrow \Rightarrow SR_{\text{Ref.}}\uparrow$

79. $\underline{SR}_{\text{Ref.}} \wedge \underline{HO}_{\text{Gov.}} \Rightarrow W_{\text{Gov.}}$

80. $\overline{SR}_{\text{Ref.}} \wedge CE_{\text{Gov.}} \Rightarrow W_{\text{Gov.}}$

81. $\overline{SR}_{Ref.} \wedge \underline{HO}_{Gov.} \wedge CE_{Gov.} \Rightarrow CO_{Gov.}\uparrow$

82. $W_{Ref.} \Rightarrow CC_{Ref.}\uparrow \vee SR_{Ref.}\uparrow$

83. $\underline{HO}_{Gov.} \wedge \underline{F}_{Gov.} \Rightarrow DC_{Ref.}$

2.2. Graph Theoretic Hypotheses with Respect to Affect Relations

84. $\underline{DC}_{Ins.}$

85. $\underline{DC}_{Inq.}$

3. Set Theoretic Hypotheses

86. $\underline{SS} \Rightarrow SA\uparrow$

87. $\overline{SE} \Rightarrow \overrightarrow{SS}$

88. $\underline{SE}\uparrow \Rightarrow SA\uparrow$

89. $SS\uparrow \Leftrightarrow SD\uparrow \ .\wedge.\ SS\downarrow \Leftrightarrow SD\downarrow$

4. Information and Graph Theoretic Hypotheses

4.1. Information Theoretic Antecedent--Graph Theoretic Consequent

90. $TP\uparrow \Rightarrow CE\downarrow$

91. $FI\downarrow \Rightarrow U\downarrow$

92. $\overline{FI}\downarrow \Rightarrow HO\downarrow$

93. $FI\downarrow \Rightarrow XD\uparrow$

94. $\overline{FO} \Rightarrow XD\uparrow$

95. $FT\uparrow \Rightarrow \overline{WE}$

96. $\overline{\overline{TP}} \wedge FP\uparrow \Rightarrow DC\uparrow$

97. $FI\uparrow \wedge \overline{\overline{CP}} \Rightarrow DC\uparrow$

98. $SP\uparrow \wedge.\ FL\downarrow \vee SL\downarrow\ .\Rightarrow IG\uparrow$

4.1.1. Information Theoretic Antecedent--Graph Theoretic Consequent with Respect to Affect Relation

99. $IP\uparrow \wedge \underline{SP} \Rightarrow SG_{Ref.}$

4.2. Graph Theoretic Antecedent--Information Theoretic Consequent

100. $CC\uparrow \Rightarrow FI\uparrow$

101. $\underline{WE} \Rightarrow \overline{FT}$

102. $ID\uparrow \Rightarrow FI\uparrow$

103. $W\uparrow \Rightarrow \overline{RG}$

104. $\underline{CO}\uparrow \Rightarrow EF\uparrow$

105. $CE\uparrow \Rightarrow TP\downarrow$

106. $CC\uparrow \vee SR\uparrow \Rightarrow TP\uparrow$

107. $CC\uparrow \vee SR\uparrow \Rightarrow IP\uparrow$

108. $CC\uparrow \vee SR\uparrow \Rightarrow FL\downarrow$

109. $CC\uparrow \vee SR\uparrow \Rightarrow SL\uparrow$

110. $CC\uparrow \vee SR\uparrow \Rightarrow 0 < \Delta FP < \Delta IP$

111. $CC\uparrow \vee SR\uparrow \Rightarrow \Delta FP < \Delta SP$

112. $SR\uparrow \wedge \overrightarrow{HO} \Rightarrow RG\downarrow$

113. $W\uparrow \wedge \overrightarrow{HO} \Rightarrow EF\downarrow$

114. $WE \wedge HO \Rightarrow F\downarrow$

115. $U \vee WE\uparrow \vee DC\uparrow \Rightarrow IP\downarrow \wedge FP\downarrow$

4.2.1. Graph Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

116. $D_{P\ Rew.}\uparrow \Rightarrow FO\downarrow$

117. $D_{P\ Rew.}\uparrow \Rightarrow \underline{AD}\uparrow$

118. $I_{Gov.}\uparrow \Rightarrow FP\uparrow$

119. $I_{\text{Gov.}}\uparrow \Rightarrow \overline{\text{OP}}$

120. $I_{\text{Gov.}}\uparrow \Rightarrow \text{FO}\downarrow$

121. $\underline{W}_{\text{Ref.}} \Rightarrow \underline{|\max \text{FP} - \text{FP}|}$

122. $\underline{W}_{\text{Ref.}} \Rightarrow \overline{\overline{\text{O}}}$

123. $\text{HO}_{\text{Gov.}}\uparrow \Rightarrow \text{FL}\uparrow$

124. $\underline{\text{CX}}_{\text{Fac.}} \Rightarrow \underline{\text{RG}}$

4.2.2. Both Graph Theoretic Antecedent and Information Theoretic Consequent with Respect to Affect Relations

125. $\underline{\text{CX}}_{\text{Fac.}} \Rightarrow \overline{\text{FT}}_{\text{Fac.}}$

4.2.3. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information Theoretic Consequent

126. $D_{P\ \text{Inq.-Leg.}}\uparrow \Rightarrow \text{FO}\uparrow \wedge \text{SL}\uparrow \wedge \underline{\max \text{SI}}$

127. $D_{P\ \text{Inq.-Exp.}}\uparrow \Rightarrow \text{FO}\downarrow \wedge \underline{\text{SL}}\uparrow \wedge \overline{\max \text{SI}}$

128. $\underline{D_A}\ \text{Fac.-Leg.} \Rightarrow \overline{\text{RG}}$

129. $W_{\text{Inq.-Ref.}}\uparrow \Rightarrow \frac{\max \text{SI}}{\text{IP}}\uparrow$

130. $\text{CC}_{\text{Ins.-Ref.}} \wedge \underline{\text{DC}}_{\text{Ins.-Ref.}}\uparrow \wedge W_{\text{Ins.-Ref.}}\uparrow \Rightarrow$

$\text{IP}\uparrow \wedge \text{FP}\uparrow \wedge \text{FO}\downarrow \wedge \text{RG}\uparrow$

131. $\text{CC}_{\text{Ins.-Exp.}} \wedge \underline{\text{DC}}_{\text{Ins.-Exp.}}\uparrow \wedge W_{\text{Ins.-Exp.}}\uparrow \Rightarrow$

$\text{IP}\uparrow \wedge \text{SP}\uparrow \wedge \text{FO}\uparrow \wedge \text{FL}\uparrow$

132. $\underline{\text{DC}}_{\text{Ins.-Ref.}} \wedge D_{P\ \text{Ins.-Ref.}}\uparrow \wedge W_{\text{Ins.-Ref.}}\uparrow \Rightarrow$

$\text{IP}\downarrow \wedge \text{FP}\downarrow \wedge \text{FO}\downarrow \wedge \text{RG}\downarrow$

133. $\underline{DC}_{\text{Ins.-Rew.}} \wedge D_{P}\ \text{Ins.-Rew.}^{\uparrow} \wedge W_{\text{Ins.-Rew.}}^{\uparrow} \Rightarrow.$

$\underline{EC}_{\overline{S}} \Rightarrow \underline{AD}\ .\wedge\ \overline{TP} \wedge \overline{SP} \wedge \underline{FL}$

134. $\underline{DC}_{\text{Ins.-Leg.}} \wedge D_{P}\ \text{Ins.-Leg.}^{\uparrow} \wedge W_{\text{Ins.-Leg.}}^{\uparrow} \Rightarrow$

$FO\uparrow \wedge \underline{SL} \wedge \underline{RG}$

135. $\underline{DC}_{\text{Ins.-Pun.}} \wedge D_{P}\ \text{Ins.-Pun.}^{\uparrow} \wedge W_{\text{Ins.-Pun.}}^{\uparrow} \wedge$

$HO_{\text{Ins.-Pun.}}^{\uparrow} \Rightarrow.\ \underline{EC}_{\overline{S}} \Rightarrow \overline{AD}\ .\wedge\ FP\downarrow \wedge FO\downarrow \wedge RG\downarrow \wedge SB\uparrow \wedge EL\uparrow$

4.3. Graph Theoretic Antecedent--Information and Graph Theoretic Consequent

4.3.1. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information and Graph Theoretic Consequent

136. $\max D_{A}\ \text{Inq.}_{\text{Dev.}}\text{-Leg.} \Rightarrow \overline{FP} \wedge FL\uparrow \wedge SL\uparrow \wedge \overline{RG} \wedge D_{A}\ \text{Ins.}^{\uparrow} \wedge$

$D_{A}\ \text{Inq.}^{\downarrow}$

4.4. Information and Graph Theoretic Antecedent--Graph Theoretic Consequent

137. $\underline{FO} \wedge \overline{CO} \Rightarrow \overline{SG}$

4.5. Information and Graph Theoretic Antecedent--Information Theoretic Consequent

138. $TP\uparrow \wedge \underline{CO}\uparrow \Rightarrow RG\uparrow$

139. $TP\uparrow \wedge {\sim}\underline{CO}\uparrow \Rightarrow EF\downarrow$

140. $F\vec{P} \vee FP\downarrow\ .\wedge\ CC\uparrow \wedge SR\uparrow .\Rightarrow FT\downarrow$

4.5.1. Information and Graph Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

141. $TP\uparrow \wedge I_{\text{Gov.}}\uparrow \Rightarrow FO\uparrow$

142. $\underline{FB} \wedge D_{P}\ \text{Pun.} \wedge \underline{D_{A}} \Rightarrow \underline{EF}$

5. Information and Set Theoretic Hypotheses

5.1. Information Theoretic Antecedent--Set Theoretic Consequent

143. $F\vec{I} \Rightarrow \overline{IIS}$

144. $FL\downarrow \Rightarrow IM\uparrow$

145. $\underline{FL} \Rightarrow \underline{SB}$

146. $\underline{AD} \Rightarrow SB\downarrow$

147. $TP\uparrow \wedge \overline{\overline{FO}} \Rightarrow SE\uparrow$

148. $\underline{EC}_{\overline{S}} \wedge \sim\underline{FT} \wedge \underline{FB} \Rightarrow \overline{SB}$

5.1.1. Both Information Theoretic Antecedent and Set Theoretic Consequent with Respect to Affect Relations

149. $FL_{Ins.}\uparrow \Rightarrow IM_{Ins.}\uparrow$

5.2. Set Theoretic Antecedent--Information Theoretic Consequent

150. $AM\uparrow \Rightarrow IP\uparrow \wedge FP\downarrow \wedge SP\uparrow \wedge FO\downarrow \wedge FL\downarrow \wedge SL\downarrow \wedge EF\downarrow$

151. $IM\uparrow \Rightarrow FP\downarrow \wedge FO\downarrow$

152. $\underline{SS} \Rightarrow \overline{AD}$

153. $SD\uparrow \Rightarrow RG\downarrow$

154. $SD\uparrow \Rightarrow SI\downarrow$

155. $\underline{EL} \Rightarrow \overline{RG}$

156. $EL(t_1) \wedge \underline{IIS} \Rightarrow \overline{RG}$

5.2.1. Set Theoretic Antecedent with Respect to Affect Relation--Information Theoretic Consequent

157. $IM_{Ins.}\uparrow \Rightarrow FP\downarrow \wedge FO\downarrow$

5.3. Information and Set Theoretic Antecedent--Information Theoretic Consequent

158. $TP\uparrow \wedge \overrightarrow{SZ} \Rightarrow FD\uparrow$

159. $\underline{EC_{\overline{S}}} \wedge \underline{CP} \wedge \underline{SB} \Rightarrow \underline{SP} \vee \underline{FL} \vee \underline{SL}$

160. $TP\uparrow \wedge FP\uparrow \wedge \overrightarrow{SZ} \Rightarrow FO\uparrow$

161. $O\vec{P} \wedge \underline{HM} \wedge AM\downarrow \Rightarrow FO\downarrow$

5.4. Information and Set Theoretic Antecedent--Set Theoretic Consequent

162. $\overline{TP} \wedge FI\uparrow \wedge \overline{SB} \Rightarrow SB\uparrow$

163. $\underline{TP} \wedge FI\downarrow \wedge \overline{SB} \Rightarrow SB\uparrow$

6. Graph and Set Theoretic Hypotheses

6.1. Graph Theoretic Antecedent--Set Theoretic Consequent

164. $I\uparrow \Rightarrow \overline{SB}$

165. $F\downarrow \Rightarrow SD\uparrow$

166. $CE\uparrow \Rightarrow SS\uparrow$

167. $\underline{CX}\uparrow \Rightarrow SZ\uparrow$

168. $I\uparrow \wedge W\uparrow \Rightarrow \underline{SS}$

169. $\underline{W} \wedge \underline{CE} \Rightarrow \underline{SD}$

6.1.1. Both Graph Theoretic Antecedent and Set Theoretic Consequent with Respect to Affect Relations

170. $CE_{Ins.}\uparrow \Rightarrow IM_{Ins.}\uparrow$

171. $\underline{DC}_{Fac.} \wedge \overline{W}_{Fac.} \Rightarrow \overline{SD}_{Fac.}$

6.2. Set Theoretic Antecedent--Graph Theoretic Consequent

172. $AM\uparrow \Rightarrow W\downarrow$

173. $AM\uparrow \Rightarrow CE\downarrow$

174. $\Delta SZ > \Delta HO$

6.3. Graph Theoretic Antecedent--Graph and Set Theoretic Consequent

175. $XD\uparrow \Rightarrow DC\uparrow \vee ZD\uparrow$

6.4. Set Theoretic Antecedent--Graph and Set Theoretic Consequent

176. $\overline{SS} \Rightarrow \overline{SG} \wedge \overline{IG} \wedge \overline{HS}$

6.5. Graph and Set Theoretic Antecedent--Graph Theoretic Consequent

177. $\max WE \wedge SZ\uparrow \Rightarrow D_P\uparrow \vee D_A\uparrow$

178. $\underline{HO}(t_1) \wedge \underline{SZ}(t_1) \Rightarrow I(t_2)\uparrow$

179. $X\vec{G} \wedge SZ\uparrow \Rightarrow VN\uparrow$

180. $X\vec{G} \wedge SZ\uparrow \Rightarrow F\downarrow$

181. $X\vec{G} \wedge SZ\uparrow \Rightarrow CE\downarrow$

182. $XD\uparrow \wedge S\vec{Z} \Rightarrow DC\uparrow$

183. $XD\uparrow \wedge SZ\downarrow \Rightarrow DC\downarrow$

184. $CX\uparrow \wedge Z\vec{G} \Rightarrow CO\downarrow$

185. $CX\uparrow \wedge Z\vec{G} \Rightarrow CE\uparrow$

6.6. Graph and Set Theoretic Antecedent--Set Theoretic Consequent

186. $CE\uparrow \wedge \underline{SE} \Rightarrow SB\downarrow$

187. $CE\uparrow \wedge SE = 0 \Rightarrow SB\uparrow$

188. $X\vec{G} \wedge SZ\uparrow \Rightarrow SD\uparrow$

7. Information, Graph, and Set Theoretic Hypotheses

7.1. Graph Theoretic Antecedent--Information and Set Theoretic Consequent

7.1.1. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information and Set Theoretic Consequent with Respect to Affect Relation

189. $\max D_{A\ \text{Inq.}_{\text{Res.}}\text{-Leg.}} \Rightarrow FP\uparrow \wedge IP\uparrow \wedge SP\uparrow \wedge FL\uparrow \wedge AM_{\text{Ins.}}\uparrow$

7.2. Set Theoretic Antecedent--Information, Graph, and Set Theoretic Consequent

190. $HM(t_2) > HM(t_1) \Rightarrow \underline{\underline{TP}} \wedge \underline{\underline{XD}} \wedge \underline{\underline{ZD}}$

7.3. Information and Graph Theoretic Antecedent--Set Theoretic Consequent

191. $\underline{EF} \wedge \underline{CO} \Rightarrow \underline{SD}$

7.4. Information and Set Theoretic Antecedent--Graph Theoretic Consequent

192. $\overrightarrow{TG} \wedge ZG\downarrow \Rightarrow XG\uparrow$

193. $\overrightarrow{TD} \wedge ZD\downarrow \Rightarrow XD\uparrow$

7.5. Graph and Set Theoretic Antecedent--Information Theoretic Consequent

194. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow TP\uparrow$

195. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow FI\downarrow$

196. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow FO\uparrow \wedge \Delta FO\downarrow$

197. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow FT\uparrow$

198. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow FB\downarrow$

199. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow RG\curvearrowright$

200. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow CP\downarrow$

201. $\overrightarrow{XG} \wedge SZ\uparrow \Rightarrow EF\curvearrowright$

ERRATA

p. 14:	2.4.2:	'$p(c_i,c_j)$' not '$p(c_i \| c_j)$'
p. 24:	l. 9:	's_1s_2, s_1s_3, and s_2s_3' not 's_1s_2, s_1s_2, and s_2s_3'
	l. 12:	'from s_1 to s_2, or from s_2 to s_1, and from s_2 to s_3.' not 'from s_1 to s_2 and from s_2 to s_3.'
p. 28:	5.3:	'2.1.1' not '3.1.1'
p. 48:	18.2:	'$S' \subset \overline{\mathcal{S}}$' not '$S' \subset S$'
	18.3:	'S' is contained in $\overline{\mathcal{S}}$' not 'S' is contained in S'
p. 54:	39.3: l. 3:	'R_A' not 'RA'
p. 69:	65:	'XD' not 'TD'
	66:	'TD' not 'ID'
p. 70:		'Relation in Literature[1]' not 'Relation in Literature'
p. 135:		Delete 1.1.
p. 136:		Insert '4.3.2. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information and Set Theoretic Consequent'
p. 137:	6.1.1:	'Relations' not 'Relation'
	7:	'Hypotheses' not 'Hopotheses'
	7.1.1:	'Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--' not 'Graph Theoretic Antecedent with Respect to Affect Relations--'
p. 138:		Delete 1.1.

p. 141:	32a:	'If school environmental changeness is greater than some value and' not 'If school environmental changeness and'
	32a and b:	Insert on p. 159 preceding 147a.
p. 143:	47a and b:	Interchange second and third conjuncts of antecedent.
p. 152:	114a and b:	Insert on p. 146 preceding 70a.
p. 154:		Insert 127a and b before 126a and b.
	129b:	'$\frac{\text{max SI}}{\text{IP}}\uparrow$' not '$\frac{\text{max SI}}{\text{IP}}$'
p. 155:	130a and b:	Interchange first and second conjuncts of antecedent.
	131a and b:	Interchange first and second conjuncts of antecedent.
p. 156:		Insert 134a and b before 133a and b.
	135b:	'$W_{\text{Ins.-Pun.}}\uparrow$' not '$W_{\text{Ins.-Leg.}}\uparrow$'
	135a and b:	Insert on p. 157 under Insertion of 4.3.2.
p. 157:	136a and b:	Interchange last two conjuncts of consequent. Insert '4.3.2. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information and Set Theoretic Consequent'
p. 159:	150a and b:	Interchange second and third conjuncts of consequent.
	151a and b:	Insert 151a and b before 150a and b.
p. 161:	161a and b:	Interchange second and third conjuncts of antecedent.
p. 162:	6.1.1:	'Relations' not 'Relation'
p. 163:	174a and b:	Insert on p. 161 under 6.
	175a and b:	Interchange alternates of consequent.

pp. 163-4:	179a and b through 183a and b:	Interchange conjuncts of antecedent.
pp. 164-5:	187a and b through 188a and b:	Interchange conjuncts of antecedent.
p. 165:	7.1.1:	'Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--' not 'Graph Theoretic Antecedent with Respect to Affect Relation--'
	190a and b:	Interchange last two conjuncts of consequent.
pp. 166-7:	192a and b through 201a and b:	Interchange conjuncts of antecedent.
p. 171:	l. 5:	'$t_2 - t_1$' not '$t_2 - t_2$'
p. 176:	l. 11:	'school' not 'cchool'
p. 180:	125a:	'affect relation' not 'affect relations'
Appendix II:	22:	In the school hypotheses, due to the use of '‾', '∼' is used for 'it is not the case that' and '≡' is used for 'is equivalent to'.
Appendix III:	IP:	Insert 50 before 10.
	FP:	Insert 96 after 33. Insert 50 before 30.
	OP:	In 161, interchange second and third conjuncts of antecedent.
	SP:	Insert 46 after 98.
	FI:	Insert 51 before 20.
	FO:	Insert 51 before 94.
	FB:	In 148, place last conjunct of antecedent first.
	FL:	Insert 46 after 98.
	SL:	Insert 46 after 98. Insert 47 before 46.

DC: Insert 84 before 58.
Insert 85 after 84.

D_p: Insert 72 after 126.
Insert 127 before 126.
Insert 71 before 132.

HO: Insert 135 after 81.
Insert 174 after 123.

SZ: Insert 174 before 177.
Insert 158 after 174.
Insert 160 after 183.

Appendix IV: 32: Insert before 147.

114: Insert before 70.

134: Insert before 133.

135: Insert under '4.3.2. Graph Theoretic Antecedent with Respect to Two Kinds of Affect Relations--Information and Set Theoretic Consequent'.

174: Insert under 6.